

Generating Equivalent Algebraic Expressions

MODULE



10

LESSON 10.1

Modeling and Writing Expressions



6.EE.2a, 6.EE.2b,
6.EE.4, 6.EE.6

LESSON 10.2

Evaluating Expressions



6.EE.2c

LESSON 10.3

Generating Equivalent Expressions



6.EE.2b, 6.EE.3,
6.EE.4



ESSENTIAL QUESTION

How can you generate equivalent algebraic expressions and use them to solve real-world problems?



Real-World Video

Carpenters use formulas to calculate a project's materials supply. Sometimes formulas can be written in different forms. The perimeter of a rectangle can be written as $P = 2(l + w)$ or $P = 2l + 2w$.

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Are YOU Ready?

Complete these exercises to review skills you will need for this module.



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Use of Parentheses

EXAMPLE $(6 + 4) \times (3 + 8 + 1) = 10 \times 12$
 $= 120$

Do the operations inside parentheses first.
Multiply.

Evaluate.

1. $11 + (20 - 13)$

2. $(10 - 7) - (14 - 12)$

3. $(4 + 17) - (16 - 9)$

4. $(23 - 15) - (18 - 13)$

5. $8 \times (4 + 5 + 7)$

6. $(2 + 3) \times (11 - 5)$

Words for Operations

EXAMPLE Write a numerical expression for the quotient of 20 and 5.

Think: *Quotient means to divide.*

$20 \div 5$

Write *20 divided by 5.*

Write a numerical expression for the word expression.

7. the difference between 42 and 19 _____

8. the product of 7 and 12 _____

9. 30 more than 20 _____

10. 100 decreased by 77 _____

Evaluate Expressions

EXAMPLE Evaluate $2(5) - 3^2$.

$2(5) - 3^2 = 2(5) - 9$
 $= 10 - 9$
 $= 1$

Evaluate exponents.
Multiply.
Subtract.

Evaluate the expression.

11. $3(8) - 15$ _____

12. $4(12) + 11$ _____

13. $3(7) - 4(2)$ _____

14. $4(2 + 3) - 12$ _____

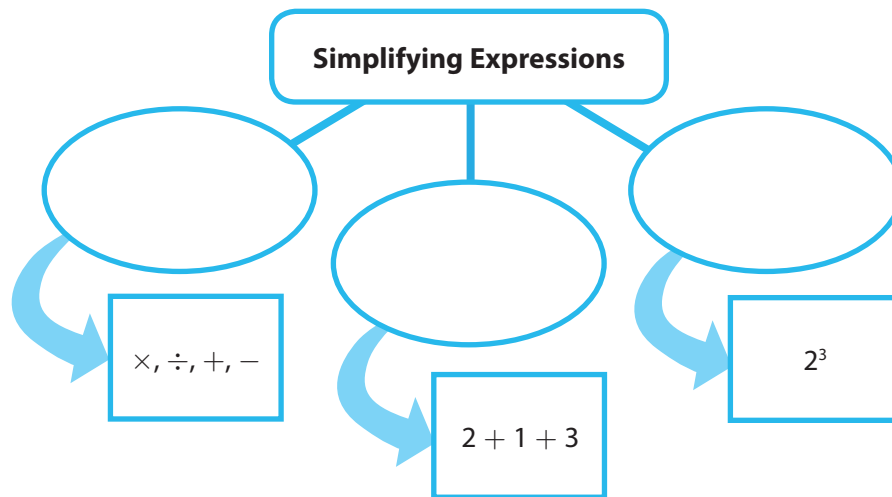
15. $9(14 - 5) - 42$ _____

16. $7(8) - 5(8)$ _____

Reading Start-Up

Visualize Vocabulary

Use the review words to complete the graphic. You may put more than one word in each oval.



Understand Vocabulary

Complete the sentences using the preview words.

1. An expression that contains at least one variable is an _____.
2. A part of an expression that is added or subtracted is a _____.
3. A _____ is a specific number whose value does not change.

Vocabulary

Review Words

base (*base*)
exponent (*exponente*)
numerical expression (*expresión numérica*)
operations (*operaciones*)
order of operations (*orden de las operaciones*)

Preview Words

algebraic expression (*expresión algebraica*)
coefficient (*coeficiente*)
constant (*constante*)
equivalent expression (*expresión equivalente*)
evaluating (*evaluar*)
like terms (*términos semejantes*)
term (*término, en una expresión*)
variable (*variable*)

Active Reading

Key-Term Fold Before beginning the module, create a key-term fold to help you learn the vocabulary in this module. Write the highlighted vocabulary words on one side of the flap. Write the definition for each word on the other side of the flap. Use the key-term fold to quiz yourself on the definitions used in this module.





Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

COMMON CORE

6.EE.2

Write, read, and evaluate expressions in which letters stand for numbers.

Key Vocabulary

expression (*expresión*)

A mathematical phrase that contains operations, numbers, and/or variables.

What It Means to You

You will use models to compare expressions.

UNPACKING EXAMPLE 6.EE.2

On a math quiz, Tina scored 3 points more than Yolanda. Juan scored 2 points more than Yolanda and earned 2 points as extra credit.

Write expressions for the numbers of points that Juan and Tina scored. Use y to represent the number of points that Yolanda scored.

$$\text{Tina's points: } y + 3$$

$$\text{Juan's points: } y + 2 + 2$$

Suppose Yolanda scored 82 points. Use the expressions to find the number of points Tina and Juan scored.

$$\text{Tina's points: } y + 3 = 82 + 3 = 85 \text{ points}$$

$$\text{Juan's points: } y + 2 + 2 = 82 + 2 + 2 = 86 \text{ points}$$

COMMON CORE

6.EE.3

Apply the properties of operations to generate equivalent expressions.

Key Vocabulary

equivalent expressions

(*expresión equivalente*)

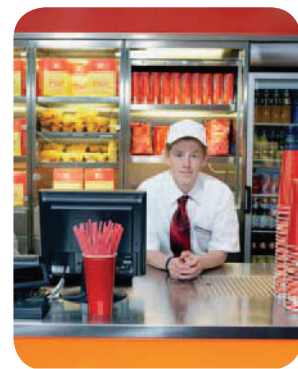
Expressions that have the same value for all values of the variables.

What It Means to You

You will use the properties of operations to find an equivalent expression.

UNPACKING EXAMPLE 6.EE.3

William earns \$13 an hour working at a movie theater. He worked h hours in concessions and three times as many hours at the ticket counter. Write and simplify an expression for the amount of money William earned.



$$\$13 \cdot \text{hours at concessions} + \$13 \cdot \text{hours at ticket counter}$$

$$13h + 13(3h)$$

$$13h + 39h$$

$$h(13 + 39)$$

$$52h$$

Multiply $13 \cdot 3h$.

Distributive Property

Simplify.



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LESSON 10.1 Modeling and Writing Expressions

COMMON CORE 6.EE.2a

Write expressions that record operations with numbers and with letters standing for numbers. Also 6.EE.2b, 6.EE.4, 6.EE.6



ESSENTIAL QUESTION

How can you model and write algebraic expressions?

Writing Algebraic Expressions

An **algebraic expression** is an expression that contains one or more variables and may also contain operation symbols, such as $+$ or $-$.

A **variable** is a letter or symbol used to represent an unknown or unspecified number. The value of a variable may change.

A **constant** is a specific number whose value does not change.

150 is a constant and y is a variable.

Algebraic Expressions	x	$w + n$	$150 + y$
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In algebraic expressions, multiplication and division are usually written without the symbols \times and \div .

- Write $3 \times n$ as $3n$, $3 \cdot n$, or $n \cdot 3$.
- Write $3 \div n$ as $\frac{3}{n}$.

There are several different ways to describe expressions with words.

Operation	Addition	Subtraction	Multiplication	Division
Words	<ul style="list-style-type: none"> • added to • plus • sum • more than 	<ul style="list-style-type: none"> • subtracted from • minus • difference • less than 	<ul style="list-style-type: none"> • times • multiplied by • product • groups of 	<ul style="list-style-type: none"> • divided by • divided into • quotient

EXAMPLE 1

COMMON CORE

6.EE.2a, 6.EE.2b

A Write each phrase as an algebraic expression.

The sum of 7 and x

The operation is addition.

The algebraic expression is $7 + x$.

The quotient of z and 3

The operation is division.

The algebraic expression is $\frac{z}{3}$.



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B Write a phrase for each expression.

$11x$


The operation is multiplication.

The product of 11 and x

$8 - y$

The operation is subtraction.

y less than 8



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YOUR TURN

Write each phrase as an algebraic expression.

1. n times 7 _____ 2. 4 minus y _____ 3. 13 added to x _____

Write a phrase for each expression.

4. $\frac{x}{12}$ _____
5. $10y$ _____
6. $c + 3$ _____

Modeling Algebraic Expressions

Algebraic expressions can also be represented with models.

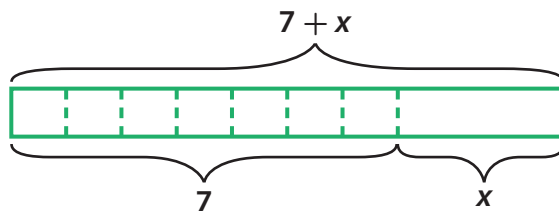
EXAMPLE 2

COMMON CORE

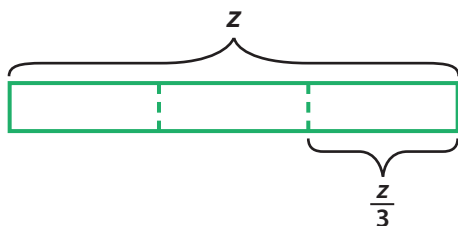
6.EE.2a

Use a bar model to represent each expression.

- A** $7 + x$ Combine 7 and x .



- B** $\frac{z}{3}$ Divide z into 3 equal parts.



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YOUR TURN

Draw a bar model to represent each expression.

7. $t - 2$

8. $4y$



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Comparing Expressions Using Models

Algebraic expressions are *equivalent* if they are equal for all values of the variable. For example, $x + 2$ and $x + 1 + 1$ are equivalent.

EXAMPLE 3

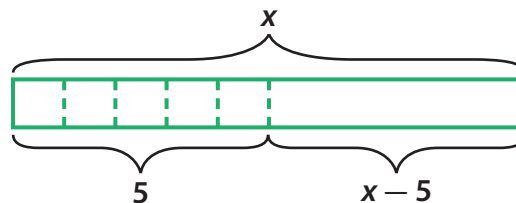


COMMON
CORE

6.EE.4

Katriana and Andrew started the day with the same amount of money. Katriana spent 5 dollars on lunch. Andrew spent 3 dollars on lunch and 2 dollars on a snack after school. Do Katriana and Andrew have the same amount of money left?

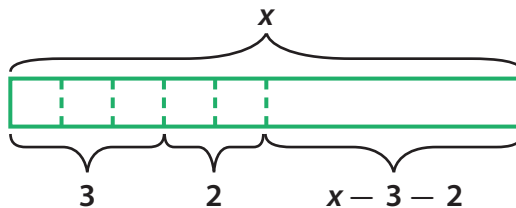
- STEP 1** Write an algebraic expression to represent the money Katriana has left. Represent the expression with a model.



$x - 5$

The variable represents the amount of money both Katriana and Andrew have at the beginning of the day.

- STEP 2** Write an algebraic expression to represent the money Andrew has left. Represent the expression with a model.



$x - 3 - 2$

- STEP 3** Compare the models.

The models are equivalent, so the expressions are equivalent.


Andrew and Katriana have the same amount of money left.



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YOUR TURN

9. On a math quiz, Tina scored 3 points more than Julia. Juan scored 2 points more than Julia and earned 2 points in extra credit. Write an expression and draw a bar model to represent Tina's score and Juan's score. Did Tina and Juan make the same grade on the quiz? Explain.



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Modeling Real-World Situations

You can use expressions to represent real-world situations.

EXAMPLE 4



COMMON CORE 6.EE.2.6

- A** Tickets to the water park cost \$53 per person. Write an expression to show the total cost of tickets for a group of people.

A group of is a clue to multiply. The ticket price of \$53 is a constant. The number of people who need tickets is a variable.

Use x for the number of people.

The algebraic expression for the total cost of tickets is $53x$.

- B** Genise has some savings. After babysitting, she adds \$75 to her savings. How much money has Genise saved?

Adds is a clue to use addition. The \$75 Genise added to her savings is a constant. The amount of money Genise had saved before is unknown, so use a variable.

Use y for Genise's savings before she adds the babysitting money.

The algebraic expression for Genise's total savings is $y + 75$.

Math Talk

Mathematical Practices

How do you know what operation to use to find the amount each niece receives?

YOUR TURN

10. Helen divides up some money to give equally to her four nieces. If d represents the total amount, write an expression to represent how much money each niece receives. _____

Guided Practice

Write each phrase as an algebraic expression. (Example 1)

1. 3 less than y _____ 2. The product of 2 and p _____

Write a phrase for each algebraic expression. (Example 1)

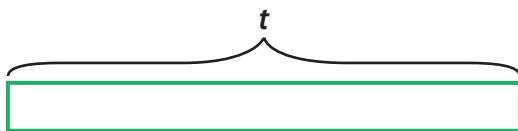
3. $y + 12$ _____ 4. $\frac{p}{10}$ _____

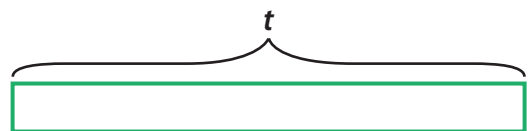
5. Draw a bar model to represent the expression $m \div 4$. (Example 2)



At 6 p.m., the temperature in Phoenix, AZ, t , is the same as the temperature in Tucson, AZ. By 9 p.m., the temperature in Phoenix has dropped 2 degrees and in Tucson it has dropped 4 degrees. By 11 p.m., the temperature in Phoenix has dropped another 3 degrees. (Example 3)

6. Represent each city's temperature at 11 p.m. with an algebraic expression and a bar model.





7. Are the expressions that represent the temperatures in the two cities equivalent? Justify your answer.

8. Noelle bought some boxes of water bottles for a picnic. Each box contained 24 bottles of water. If c is the number of boxes, write an expression to show how many bottles of water Noelle bought. (Example 4)

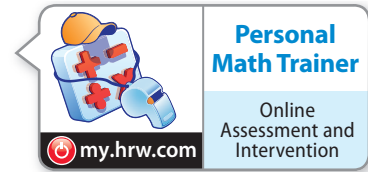


ESSENTIAL QUESTION CHECK-IN

9. Give an example of a real-world situation that could be represented by an algebraic expression.

10.1 Independent Practice

COMMON CORE 6.EE.2a, 6.EE.2b, 6.EE.4, 6.EE.2.6



10. Write an algebraic expression with the constant 7 and the variable y .

Write each phrase as an algebraic expression.

- 11.** n divided by 8 _____
- 12.** p multiplied by 4 _____
- 13.** b plus 14 _____
- 14.** 90 times x _____
- 15.** a take away 16 _____
- 16.** k less than 24 _____
- 17.** 3 groups of w _____
- 18.** the sum of 1 and q _____
- 19.** the quotient of 13 and z _____
- 20.** c added to 45 _____
- 21.** 8 less than w _____

Write a phrase in words for each algebraic expression.

- 22.** $m + 83$ _____
- 23.** $42s$ _____
- 24.** $\frac{9}{d}$ _____
- 25.** $t - 29$ _____
- 26.** $2 + g$ _____
- 27.** $11x$ _____
- 28.** $\frac{h}{12}$ _____
- 29.** $5 - k$ _____

Sarah and Noah work at Read On Bookstore and get paid the same hourly wage. The table shows their work schedule for last week.

Read On Bookstore Work Schedule (hours)			
	Monday	Tuesday	Wednesday
Sarah	5	3	
Noah			8

- 30.** Write an expression that represents Sarah's total pay last week. Represent her hourly wage with w . _____
- 31.** Write an expression that represents Noah's total pay last week. Represent his hourly wage with w . _____
- 32.** Are the expressions equivalent? Did Sarah and Noah earn the same amount last week? Use models to justify your answer.

- 33.** Mia buys 3 gallons of gas that costs d dollars per gallon. Bob buys g gallons of gas that costs \$3 per gallon.
 - a.** Write an expression for the amount Mia pays for gas. _____
 - b.** Write an expression for the amount Bob pays for gas. _____
 - c.** What do the numeral and the variable represent in each expression?

- 34.** The student council is asking people to donate money for the new park outside the school. Everyone who makes the suggested donation amount will be given a bracelet. If everyone donates the suggested amount, and b bracelets are given away, what algebraic expression represents the total amount collected in donations?
-



- 35.** Mr. Delgado has some young orange trees. He wants to plant them in 46 rows. If t is the total number of orange trees, write an algebraic expression to represent how many trees he can plant in each row.
-

- 36.** There are 15 violinists in the orchestra this year. Next year, two violinists will leave and some new violinists will join the orchestra. If v is the number of violinists who will join the orchestra, write an expression to represent the number of violinists in the orchestra next year.
-

- 37.** Jill, Meg, and Beth are sisters. Jill is 2 years younger than Meg. Beth is half as old as Meg. Let m represent Meg's age. Write two other algebraic expressions based on this situation. Tell what each expression represents, and what the variable stands for in each expression.
-
-

- 38. Multistep** Will, Hector, and Lydia volunteered at the animal shelter in March and April. The table shows the number of hours Will and Hector volunteered in March. Let x represent the number of hours Lydia volunteered in March.

March Volunteering	
Will	3 hours
Hector	5 hours

- a.** Will's volunteer hours in April were equal to his March volunteer hours plus Lydia's March volunteer hours. Write an expression to represent Will's volunteer hours in April.
-
- b.** Hector's volunteer hours in April were equal to 2 hours less than his March volunteer hours plus Lydia's March volunteer hours. Write an expression to represent Hector's volunteer hours in April.
-
- c.** Did Will and Hector volunteer the same number of hours in April?
Explain. _____

- 39.** The town of Rayburn received 6 more inches of snow than the town of Greenville. Let g represent the amount of snow in Greenville. Write an algebraic expression to represent the amount of snow in Rayburn.
-

40. Abby baked 48 dinner rolls and divided them evenly into bags. Let b represent the number of bags. Write an algebraic expression to represent the number of dinner rolls in each bag.

41. Eli is driving at a speed of 55 miles per hour. Let h represent the number of hours that Eli drives at this speed. Write an algebraic expression to represent the number of miles that Eli travels during this time.

H.O.T. FOCUS ON HIGHER ORDER THINKING

42. **Multistep** Bob's Bagels offers two breakfast options, as shown.



a. Let x represent the number of customers who order coffee and a bagel. How much money will Bob's Bagels make from these orders?

b. Let y represent the number of customers who order tea and a breakfast sandwich. How much money will Bob's Bagels make from these orders?

c. Write an algebraic expression for the total amount Bob's Bagels will make from all the coffee and bagel orders and from all the tea and breakfast sandwich orders.

43. **Represent Real-World Problems** The number of shoes in a closet is s .

a. How many pairs of shoes are in the closet? Explain.

b. **What If?** Suppose one of the pairs is missing a shoe. How many shoes are in the closet?

44. **Problem Solving** Write an expression that has three terms, two different variables, and one constant.

45. **Represent Real-World Problems** Describe a situation that can be modeled by the expression $x - 8$.

46. **Critique Reasoning** Ricardo says that the expression $y + 4$ is equivalent to the expression $1y + 4$. Is he correct? Explain.

Work Area

LESSON 10.2 Evaluating Expressions

COMMON CORE 6.EE.2c

Evaluate expressions at specific values of their variables.



ESSENTIAL QUESTION

How can you use the order of operations to evaluate algebraic expressions?

Evaluating Expressions

Recall that an algebraic expression contains one or more variables. You can substitute a number for that variable and then find the value of the expression. This is called **evaluating** the expression.



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EXAMPLE 1

COMMON CORE 6.EE.2c

Evaluate each expression for the given value of the variable.

A $x - 9; x = 15$

$15 - 9$ *Substitute 15 for x .*

6 *Subtract.*

When $x = 15$, $x - 9 = 6$.

B $\frac{16}{n}; n = 8$

$\frac{16}{8}$ *Substitute 8 for n .*

2 *Divide.*

When $n = 8$, $\frac{16}{n} = 2$.

C $0.5y; y = 1.4$

$0.5(1.4)$ *Substitute 1.4 for y .*

0.7 *Multiply.*

When $y = 1.4$, $0.5y = 0.7$.

D $6k; k = \frac{1}{3}$

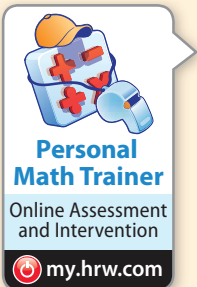
HINT: Think of 6 as $\frac{6}{1}$.

$6\left(\frac{1}{3}\right)$ *Substitute $\frac{1}{3}$ for k .*

2 *Multiply.*

When $k = \frac{1}{3}$, $6k = 2$.

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YOUR TURN

Evaluate each expression for the given value of the variable.

1. $4x$; $x = 8$ _____ 2. $6.5 - n$; $n = 1.8$ _____ 3. $\frac{m}{6}$; $m = 18$ _____

Using the Order of Operations

Expressions may have more than one operation or more than one variable. To evaluate these expressions, substitute the given value for each variable and then use the order of operations.

EXAMPLE 2

COMMON
CORE

6.EE.2c

Evaluate each expression for the given value of the variable.

A $4(x - 4)$; $x = 7$

$4(7 - 4)$ *Substitute 7 for x .*

$4(3)$ *Subtract inside the parentheses.*

12 *Multiply.*

When $x = 7$, $4(x - 4) = 12$.

B $4x - 4$; $x = 7$

$4(7) - 4$ *Substitute 7 for x .*

$28 - 4$ *Multiply.*

24 *Subtract.*

When $x = 7$, $4x - 4 = 24$.

C $w - x + y$; $w = 6$, $x = 5$, $y = 3$

$(6) - (5) + (3)$ *Substitute 6 for w , 5 for x , and 3 for y .*

$1 + 3$ *Subtract.*

4 *Add.*

When $w = 6$, $x = 5$, $y = 3$, $w - x + y = 4$.

D $x^2 - x$; $x = 9$

$(9)^2 - (9)$ *Substitute 9 for each x .*

$81 - 9$ *Evaluate exponents.*

72 *Subtract.*

When $x = 9$, $x^2 - x = 72$.

Math Talk

Mathematical Practices

Is $w - x + y$ equivalent to $w - y + x$? Explain any difference in the order the math operations are performed.

YOUR TURN

Evaluate each expression for $n = 5$.

4. $3(n + 1)$ _____ 5. $4(n - 4) + 14$ _____ 6. $6n + n^2$ _____

Evaluate each expression for $a = 3$, $b = 4$, and $c = 6$.

7. $ab - c$ _____ 8. $bc + 5a$ _____ 9. $a^3 - (b + c)$ _____

Evaluating Real-World Expressions

You can evaluate expressions to solve real-world problems.

EXAMPLE 3



COMMON CORE 6.EE.2c

The expression $1.8c + 32$ gives the temperature in degrees Fahrenheit for a given temperature in degrees Celsius c . Find the temperature in degrees Fahrenheit that is equivalent to 30°C .

STEP 1 Find the value of c .

$$c = 30^\circ\text{C}$$

STEP 2 Substitute the value into the expression.

$$1.8c + 32$$

$$1.8(30) + 32 \quad \text{Substitute 30 for } c.$$

$$54 + 32 \quad \text{Multiply.}$$

$$86 \quad \text{Add.}$$

86°F is equivalent to 30°C .

YOUR TURN

10. The expression $6x^2$ gives the surface area of a cube, and the expression x^3 gives the volume of a cube, where x is the length of one side of the cube. Find the surface area and the volume of a cube with a side length of 2 m.

$$S = \text{_____ } m^2; V = \text{_____ } m^3$$

11. The expression $60m$ gives the number of seconds in m minutes. How many seconds are there in 7 minutes?

$$\text{_____ seconds}$$



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Guided Practice

Evaluate each expression for the given value(s) of the variable(s).

(Examples 1 and 2)

- $x - 7$; $x = 23$ _____
- $3a - b$; $a = 4$, $b = 6$ _____
- $\frac{8}{t}$; $t = 4$ _____
- $9 + m$; $m = 1.5$ _____
- $\frac{1}{2}w + 2$; $w = \frac{1}{9}$ _____
- $5(6.2 + z)$; $z = 3.8$ _____

7. The table shows the prices for games in Bella's soccer league. Her parents and grandmother attended a soccer game. How much did they spend if they all went together in one car?

Women's Soccer Game Prices	
Student tickets	\$6
Nonstudent tickets	\$12
Parking	\$5

(Example 3)

- a. Write an expression that represents the cost of one carful of nonstudent soccer fans. Use x as the number of people who rode in the car and attended the game.

_____ is an expression that represents the cost of one carful of nonstudent soccer fans.

- b. Since there are three attendees, evaluate the expression $12x + 5$ for $x = 3$.

$$12(\underline{\quad}) + 5 = \underline{\quad} + 5 = \underline{\quad}$$

The family spent _____ to attend the game.

8. Stan wants to add trim all around the edge of a rectangular tablecloth that measures 5 feet long by 7 feet wide. The perimeter of the rectangular tablecloth is twice the length added to twice the width. How much trim does Stan need to buy? (Example 3)

- a. Write an expression that represents the perimeter of the rectangular tablecloth. Let l represent the length of the tablecloth and w

represent its width. The expression would be _____.

- b. Evaluate the expression $P = 2w + 2l$ for $l = 5$ and $w = 7$.

$$2(\underline{\quad}) + 2(\underline{\quad}) = 14 + \underline{\quad} = \underline{\quad}$$

Stan bought _____ of trim to sew onto the tablecloth.

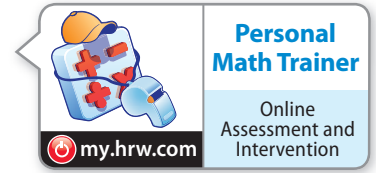


ESSENTIAL QUESTION CHECK-IN

9. How do you know the correct order in which to evaluate algebraic expressions?

10.2 Independent Practice

COMMON CORE 6.EE.2c



- 10.** The table shows ticket prices at the Movie 16 theater. Let a represent the number of adult tickets, c the number of children's tickets, and s the number of senior citizen tickets.

Movie 16 Ticket Prices	
Adults	\$8.75
Children	\$6.50
Seniors	\$6.50

- a.** Write an expression for the total cost of the three types of tickets.
- _____
- _____
- b.** The Andrews family bought 2 adult tickets, 3 children's tickets, and 1 senior ticket. Evaluate your expression in part a to find the total cost of the tickets.
- _____
- _____
- c.** The Spencer family bought 4 adult tickets and 2 children's tickets. Did they spend the same as the Andrews family? Explain.
- _____

- 11.** The area of a triangular sail is given by the expression $\frac{1}{2}bh$, where b is the length of the base and h is the height. What is the area of a triangular sail in a model sailboat when $b = 12$ inches and $h = 7$ inches?

$A = \underline{\hspace{2cm}} \text{ in.}^2$

- 12.** Ramon wants to balance his checking account. He has \$2,340 in the account. He writes a check for \$140. He deposits a check for \$268. How much does Ramon have left in his checking account? _____

- 13. Look for a Pattern** Evaluate the expression $6x - x^2$ for $x = 0, 1, 2, 3, 4, 5,$ and 6 . Use your results to fill in the table and describe any pattern that you see.

x	0	1	2	3	4	5	6
$6x - x^2$							

- 14.** The kinetic energy (in joules) of a moving object can be calculated from the expression $\frac{1}{2}mv^2$, where m is the mass of the object in kilograms and v is its speed in meters per second. Find the kinetic energy of a 0.145-kg baseball that is thrown at a speed of 40 meters per second.

$E = \underline{\hspace{2cm}} \text{ joules}$

- 15.** The area of a square is given by x^2 , where x is the length of one side. Mary's original garden was in the shape of a square. She has decided to double the area of her garden. Write an expression that represents the area of Mary's new garden. Evaluate the expression if the side length of Mary's original garden was 8 feet.

16. The volume of a pyramid with a square base is given by the expression $\frac{1}{3}s^2h$, where s is the length of a side of the base and h is the height. Find the volume of a pyramid with a square base of side length 24 feet and a height of 30 feet.



FOCUS ON HIGHER ORDER THINKING

17. **Draw Conclusions** Consider the expressions $3x(x - 2) + 2$ and $2x^2 + 3x - 12$.
- a. Evaluate each expression for $x = 2$ and for $x = 7$. Based on your results, do you know whether the two expressions are equivalent? Explain.

- b. Evaluate each expression for $x = 5$. Based on your results, do you know whether the two expressions are equivalent? Explain.

18. **Critique Reasoning** Marjorie evaluated the expression $3x + 2$ for $x = 5$ as shown:

$$3x + 2 = 35 + 2 = 37$$

What was Marjorie's mistake? What is the correct value of $3x + 2$ for $x = 5$?

Work Area

LESSON 10.3 Generating Equivalent Expressions

COMMON CORE 6.EE.3

Apply the properties of operations to generate equivalent expressions. Also 6.EE.2b, 6.EE.4



ESSENTIAL QUESTION

How can you identify and write equivalent expressions?

EXPLORE ACTIVITY 1

COMMON CORE 6.EE.4

Identifying Equivalent Expressions

One way to test whether two expressions might be equivalent is to evaluate them for the same value of the variable.

Match the expressions in List A with their equivalent expressions in List B.

List A

$$5x + 65$$

$$5(x + 1)$$

$$1 + 5x$$

List B

$$5x + 1$$

$$5x + 5$$

$$5(13 + x)$$

- A** Evaluate each of the expressions in the lists for $x = 3$.

List A

$$5(3) + 65 = \square$$

$$5(3 + 1) = \square$$

$$1 + 5(3) = \square$$

List B

$$5(3) + 1 = \square$$

$$5(3) + 5 = \square$$

$$5(13 + 3) = \square$$

- B** Which pair(s) of expressions have the same value for $x = 3$?

- C** How could you further test whether the expressions in each pair are equivalent?

- D** Do you think the expressions in each pair are equivalent? Why or why not?

EXPLORE ACTIVITY 1 (cont'd)

Reflect

1. **Error Analysis** Lisa evaluated the expressions $2x$ and x^2 for $x = 2$ and found that both expressions were equal to 4. Lisa concluded that $2x$ and x^2 are equivalent expressions. How could you show Lisa that she is incorrect?

EXPLORE ACTIVITY 2

COMMON CORE 6.EE.3

Modeling Equivalent Expressions

You can also use models to determine if two expressions are equivalent. *Algebra tiles* are one way to model expressions.

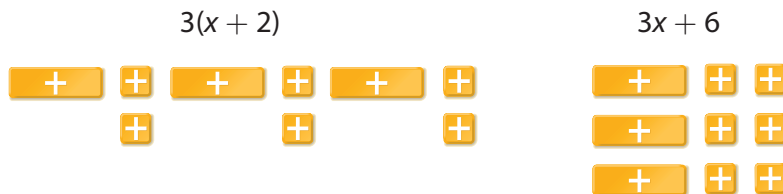
Algebra Tiles

 = 1

 = x

Determine if the expression $3(x + 2)$ is equivalent to $3x + 6$.

- A** Model each expression using algebra tiles.



- B** The model for $3(x + 2)$ has _____ x tiles and _____ 1 tiles.

The model for $3x + 6$ has _____ x tiles and _____ 1 tiles.

- C** Is the expression $3(x + 2)$ equivalent to $3x + 6$? Explain.

Reflect

2. Use algebra tiles to determine if $2(x + 3)$ is equivalent to $2x + 3$. Explain your answer.

Writing Equivalent Expressions Using Properties



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Properties of operations can be used to identify equivalent expressions.

Properties of Operations	Examples
Commutative Property of Addition: When adding, changing the order of the numbers does not change the sum.	$3 + 4 = 4 + 3$
Commutative Property of Multiplication: When multiplying, changing the order of the numbers does not change the product.	$2 \times 4 = 4 \times 2$
Associative Property of Addition: When adding more than two numbers, the grouping of the numbers does not change the sum.	$(3 + 4) + 5 = 3 + (4 + 5)$
Associative Property of Multiplication: When multiplying more than two numbers, the grouping of the numbers does not change the product.	$(2 \times 4) \times 3 = 2 \times (4 \times 3)$
Distributive Property: Multiplying a number by a sum or difference is the same as multiplying by each number in the sum or difference and then adding or subtracting.	$6(2 + 4) = 6(2) + 6(4)$ $8(5 - 3) = 8(5) - 8(3)$
Identity Property of Addition: Adding zero to a number does not change its value.	$9 + 0 = 9$
Identity Property of Multiplication: Multiplying a number by one does not change its value.	$1 \times 7 = 7$

EXAMPLE 1

COMMON CORE 6.EE.3

Use a property to write an expression that is equivalent to $x + 3$.

The operation in the expression is addition.

You can use the Commutative Property of Addition to write an equivalent expression: $x + 3 = 3 + x$.

YOUR TURN

For each expression, use a property to write an equivalent expression. Tell which property you used.

3. $(ab)c =$ _____

4. $3y + 4y =$ _____

5. 6×7 _____



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Identifying Equivalent Expressions Using Properties



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EXAMPLE 2

COMMON CORE

6.EE.3

Use the properties of operations to determine if the expressions are equivalent.

A $3(x - 2); 3x - 6$

$$3(x - 2) = 3x - 6$$

Distributive Property

$3(x - 2)$ and $3x - 6$ are equivalent expressions.

B $2 + x; \frac{1}{2}(4 + x)$

$$\frac{1}{2}(x + 4) = \frac{1}{2}x + 2$$

Distributive Property

$$= 2 + \frac{1}{2}x$$

Commutative Property

$2 + x$ does not equal $2 + \frac{1}{2}x$.

They are not equivalent expressions.

YOUR TURN

Use the properties of operations to determine if the expressions are equivalent.

6. $6x - 8; 2(3x - 5)$

7. $2 - 2 + 5x; 5x$

8. Jamal bought 2 packs of stickers and 8 individual stickers. Use x to represent the number of stickers in a pack of stickers and write an expression to represent the number of stickers Jamal bought. Is the expression equivalent to $2(4 + x)$? Check your answer with algebra tile models.



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Generating Equivalent Expressions



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Parts of an algebraic expression		
terms	The parts of the expression that are separated by + or - signs	$12 + 3y^2 + 4x + 2y^2 + 4$
coefficients	Numbers that are multiplied by at least one variable	$12 + 3y^2 + 4x + 2y^2 + 4$
like terms	Terms with the same variable(s) raised to the same power(s)	$12 + 3y^2 + 4x + 2y^2 + 4$

When an expression contains like terms, you can use properties to combine the like terms and write an equivalent expression.

EXAMPLE 3

COMMON CORE

6.EE.3, 6.EE.2b

Combine like terms.

A $6x^2 - 4x^2$

$6x^2$ and $4x^2$ are like terms.

$$6x^2 - 4x^2 = x^2(6 - 4)$$

Distributive Property

$$= x^2(2)$$

Subtract inside the parentheses.

$$= 2x^2$$

Commutative Property of Multiplication

$$6x^2 - 4x^2 = 2x^2$$

B $3a + 2(b + 5a)$

$$3a + 2(b + 5a) = 3a + 2b + 2(5a)$$

Distributive Property

$$= 3a + 2b + (2 \cdot 5)a$$

Associative Property of Multiplication

$$= 3a + 2b + 10a$$

Multiply 2 and 5.

$$= 3a + 10a + 2b$$

Commutative Property of Addition

$$= (3 + 10)a + 2b$$

Distributive Property

$$= 13a + 2b$$

Add inside the parentheses.

$$3a + 2(b + 5a) = 13a + 2b$$

C $y + 11x + 7y - 7x$

y and $7y$ are like terms;
 $11x$ and $7x$ are like terms.

$$y + 11x + 7y - 7x = y + 7y + 11x - 7x$$

Commutative Property

$$= (1 + 7)y + (11 - 7)x$$

Distributive Property

$$= 8y + 4x$$

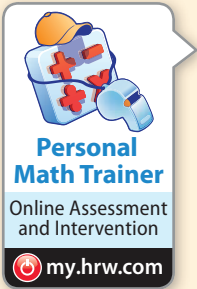
Simplify inside parentheses.

$$y + 11x + 7y - 7x = 8y + 4x$$

Math Talk

Mathematical Practices

Write 2 terms that can be combined with $7y^4$.



YOUR TURN

Combine like terms.

9. $8y - 3y =$ _____

10. $6x^2 + 4(x^2 - 1) =$ _____

11. $4a^5 - 2a^5 + 4b + b =$

12. $8m + 14 - 12 + 4n =$

Guided Practice

1. Evaluate each of the expressions in the list for $y = 5$. Then, draw lines to match the expressions in List A with their equivalent expressions in List B. (Explore Activity 1)

List A

$4 + 4y =$ _____

$4(y - 1) =$ _____

$4y + 1 =$ _____

List B

$4y - 4 =$ _____

$4(y + 1) =$ _____

$1 + 4y =$ _____

2. Determine if the expressions are equivalent by comparing the models. (Explore Activity 2) _____



For each expression, use a property to write an equivalent expression. Tell which property you used. (Example 1)

3. $ab =$ _____

4. $5(3x - 2) =$ _____

Use the properties of operations to determine if each pair of expressions is equivalent. (Example 2)

5. $\frac{1}{2}(4 - 2x); 2 - 2x$ _____

6. $\frac{1}{2}(6x - 2); 3 - x$ _____

Combine like terms. (Example 3)

7. $32y + 12y =$ _____

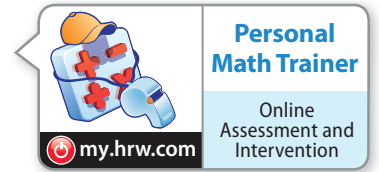
8. $12 + 3x - x - 12 =$ _____

ESSENTIAL QUESTION CHECK-IN

9. Describe two ways to write equivalent algebraic expressions.

10.3 Independent Practice

COMMON CORE 6.EE.2b, 6.EE.3, 6.EE.4



For each expression, use a property to write an equivalent expression. Tell which property you used.

10. $cd =$ _____ 11. $x + 13 =$ _____

12. $4(2x - 3) =$ _____ 13. $2 + (a + b) =$ _____

14. Draw algebra tile models to prove that $4 + 8x$ and $4(2x + 1)$ are equivalent.

Combine like terms.

15. $7x^4 - 5x^4 =$ _____ 16. $32y + 5y =$ _____

17. $6b + 7b - 10 =$ _____ 18. $2x + 3x + 4 =$ _____

19. $y + 4 + 3(y + 2) =$ _____ 20. $7a^2 - a^2 + 16 =$ _____

21. $3y^2 + 3(4y^2 - 2) =$ _____ 22. $z^2 + z + 4z^3 + 4z^2 =$ _____

23. $0.5(x^4 - 3) + 12 =$ _____ 24. $\frac{1}{4}(16 + 4p) =$ _____

25. **Justify Reasoning** Determine whether $3x + 12 + x$ is equivalent to $4(3 + x)$. Use properties of operations to justify your answer.

26. William earns \$13 an hour working at a movie theater. Last week he worked h hours at the concession stand and three times as many hours at the ticket counter. Write and simplify an expression for the amount of money William earned last week.

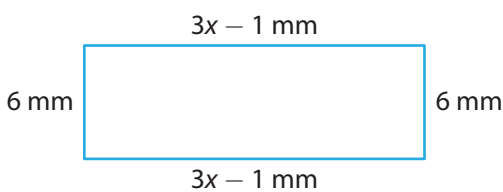
- 27. Multiple Representations** Use the information in the table to write and simplify an expression to find the total weight of the medals won by the top medal-winning nations in the 2012 London Olympic Games. The three types of medals have different weights.

2012 Summer Olympics			
	Gold	Silver	Bronze
United States	46	29	29
China	38	27	23
Great Britain	29	17	19

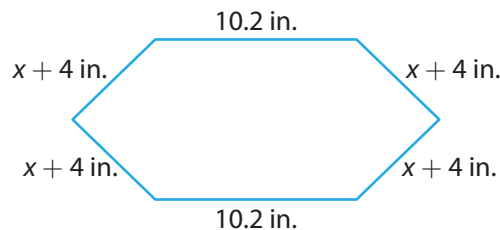


Write an expression for the perimeters of each given figure. Simplify the expression.

28. _____



29. _____



FOCUS ON HIGHER ORDER THINKING

- 30. Problem Solving** Examine the algebra tile model.

a. Write two equivalent expressions for the model. _____

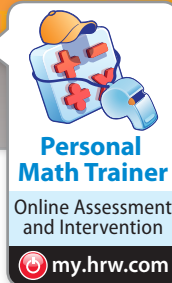
b. **What If?** Suppose a third row of tiles identical to the ones above is added to the model. How does that change the two expressions?

- 31. Communicate Mathematical Ideas** Write an example of an expression that cannot be simplified, and explain how you know that it cannot be simplified.

- 32. Problem Solving** Write an expression that is equivalent to $8(2y + 4)$ that can be simplified.

Work Area

Ready to Go On?



10.1 Modeling and Writing Expressions

Write each phrase as an algebraic expression.

1. p divided by 6 _____
2. 65 less than j _____
3. the sum of 185 and h _____
4. the product of 16 and g _____
5. Let x represent the number of television show episodes that are taped in a season. Write an expression for the number of episodes taped in 4 seasons. _____

10.2 Evaluating Expressions

Evaluate each expression for the given value of the variable.

6. $8p$; $p = 9$ _____
7. $11 + r$; $r = 7$ _____
8. $4(d + 7)$; $d = 2$ _____
9. $\frac{60}{m}$; $m = 5$ _____
10. To find the area of a triangle, you can use the expression $b \times h \div 2$, where b is the base of the triangle and h is its height. What is the area of a triangle with a base of 6 and a height of 8? _____

10.3 Generating Equivalent Expressions

11. Draw lines to match the expressions in List A with their equivalent expressions in List B.

List A	List B
$7x + 14$	$7(1 + x)$
$7 + 7x$	$7x - 7$
$7(x - 1)$	$7(x + 2)$

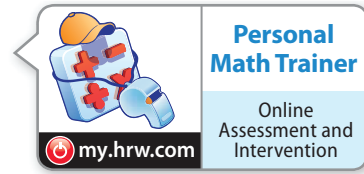


ESSENTIAL QUESTION

12. How can you solve problems involving equivalent expressions?



Assessment Readiness



Selected Response

- Which expression represents the product of 83 and x ?
(A) $83 + x$
(B) $83 \div x$
(C) $83x$
(D) $83 - x$
- Which phrase describes the algebraic expression $\frac{r}{9}$?
(A) the product of r and 9
(B) the quotient of r and 9
(C) 9 less than r
(D) r more than 9
- Rhonda was organizing photos in a photo album. She took 60 photos and divided them evenly among p pages. Which algebraic expression represents the number of photos on each page?
(A) $p - 60$ (C) $\frac{p}{60}$
(B) $60 - p$ (D) $\frac{60}{p}$
- Using the algebraic expression $4n + 6$, what is the greatest whole-number value of n that will give you a result less than 100?
(A) 22 (C) 24
(B) 23 (D) 25
- Evaluate $7w - 14$ for $w = 9$.
(A) 2
(B) 18
(C) 49
(D) 77

- Katie has read 32% of a book. If she has read 80 pages, how many more pages does Katie have left to read?
(A) 40
(B) 170
(C) 200
(D) 250
- The expression $12(x + 4)$ represents the total number of CDs Mei bought in April and May at \$12 each. Which property is applied to write the equivalent expression $12x + 48$?
(A) Associative Property of Addition
(B) Associative Property of Multiplication
(C) Commutative Property of Multiplication
(D) Distributive Property

Mini-Task

- You can convert a temperature given in degrees Celsius to a Fahrenheit temperature by using the expression $9x \div 5 + 32$, where x is the Celsius temperature.
 - Water freezes when the temperature is 0°C . At what Fahrenheit temperature does water freeze? _____
 - Water boils at 100°C . At what temperature does water boil in degrees Fahrenheit? _____
 - The temperature of some water is 15°C . What is the Fahrenheit temperature? _____