## Equations and Relationships



ESSENTIAL QUESTION
How can you use equations and relationships to solve real-world problems?


LESSON 11.1
Writing Equations to Represent Situations
6.EE.5, 6.EE.6,
6.EE. 7

LESSON 11.2
Addition and Subtraction Equations

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6.EE.5, 6.EE.6,
6.EE. 7

## LESSON 11.3

Writing Equations from Tables
6.EE.5, 6.EE.6,
6.EE. 7

LESSON 11.4
Writing Inequalities
COMMON
6.EE.5, 6.EE.6,
6.EE. 8
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## Are

Complete these exercises to review skills you will need for this module.

## Evaluate Expressions

EXAMPLE Evaluate $8(3+2)-5^{2}$

$$
\begin{aligned}
8(3+2)-5^{2} & =8(5)-5^{2} & & \text { Perform operations inside parentheses first. } \\
& =8(5)-25 & & \text { Evaluate exponents. } \\
& =40-25 & & \text { Multiply. } \\
& =15 & & \text { Subtract. }
\end{aligned}
$$

## Evaluate the expression.

1. $4(5+6)-15$ $\qquad$ 2. $8(2+4)+16$ $\qquad$
2. $3(14-7)-16$ $\qquad$ 4. $6(8-3)+3(7-4)$ $\qquad$
3. $10(6-5)-3(9-6)$ $\qquad$ 6. $7(4+5+2)-6(3+5)$ $\qquad$
4. $7(14-8)-6^{2}$ $\qquad$
5. $8(2+1)^{2}-4^{2}$ $\qquad$

## Connect Words and Equations

EXAIMPLE The product of a number and 4 is 32 .
The product of $x$ and 4 is 32 . Represent the unknown with a variable.
$4 \times x$ is 32 . Determine the operation.
$4 \times x=32$.
Determine the placement of the equal sign.
Write an algebraic equation for the word sentence.
10. A number increased by 7.9 is 8.3 . $\qquad$
12. The quotient of a number and 8 is 4 . $\qquad$
14. The difference between 31 and a number is 7 .
11. 17 is the sum of a number and 6. $\qquad$
13. 81 is three times a number.
$\qquad$
15. Eight less than a number is 19.

## Reading Start-Up

## Visualize Vocabulary

## Use the $\checkmark$ words to complete the graphic.



## Understand Vocabulary

Match the term on the left to the correct expression on the right.

1. algebraic expression
2. equation
3. solution
A. A mathematical statement that two expressions are equal.
B. A value of the variable that makes the statement true.
C. A mathematical statement that includes one or more variables.

## Active Reading

Booklet Before beginning the module, create a booklet to help you learn the concepts in this module. Write the main idea of each lesson on each page of the booklet. As you study each lesson, write important details that support the main idea, such as vocabulary and formulas. Refer to your finished booklet as you work on assignments and study for tests.

MODULE 11

# Unpocking the Stondords 

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

## 6.EE. 7

Solve real-world and mathematical problems by writing and solving equations of the form $x+p=q$ and $p x=q$ for cases in which $p, q$ and $x$ are all nonnegative rational numbers.

## Key Vocabulary

equation (ecuación)
A mathematical sentence that shows that two expressions are equivalent.

## What It Means to You

You will learn to write an equation to represent a situation.

## UNPACKING

EXAMPLE 6.EE. 7


The Falcons won their football game with a score of 30 to 19 . Kevin scored 12 points for the Falcons. Write an equation to determine how many points Kevin's teammates scored.


## 6.EE. 5

Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

## Key Vocabulary

inequality (desigualdad)
A mathematical sentence that shows the relationship between quantities that are not equal.

## What It Means to You

You can substitute a given value for the variable in an equation or inequality to check if that value makes the equation or inequality true.

## UNPACKING EXAMPLE 6.EE. 5

Melanie has to buy 6 tickets to a play. She will pay at least \$156 depending on the price of the seats. Write an inequality for this situation. Use it to decide if $\$ 20$ and $\$ 30$ are possible ticket prices.

| Number of <br> tickets bought | .Price per <br> ticket | $\geq$Total <br> cost |  |
| :---: | :---: | :---: | :---: |
| 6 | $\cdot$ | $p$ | $\geq$ |

Substitute 20 and 30 for $p$ to see if the inequality is true.
$6 p \geq 156$
$6 p \geq 156$
$6 \cdot 20 \stackrel{?}{\geq} 156$
$120 \stackrel{?}{\geq} 156 x$
$6 \cdot 30 \stackrel{?}{\geq} 156$
$180 \stackrel{?}{\geq} 156 \checkmark$

The price per ticket could be $\$ 30$ but not $\$ 20$.

How do you write equations and determine whether a number is a solution of an equation?

## Determining Whether Values Are Solutions

An equation is a mathematical statement that two expressions are equal. An equation may or may not contain variables. For an equation that has a variable, a solution of the equation is a value of the variable that makes


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(C) my.hrw.com the equation true.


## An equation represents a relationship between two

 values.

Equation
$5+4=9$
A number plus 4 is 9 .
$n+4=9$

An equation relates two expressions using symbols for is or equals.

## EXAMPLE 1

Determine whether the given value is a solution of the equation.
A $x+9=15 ; x=6$
$6+9 \stackrel{?}{=} 15$ Substitute 6 for $x$.

$$
15 \stackrel{?}{=} 15 \quad \text { Add. }
$$

6 is a solution of $x+9=15$.
B $\frac{y}{4}=32 ; y=8$
$\frac{8}{4} \stackrel{?}{=} 32 \quad$ Substitute 8 for $y$.
$2 \stackrel{?}{=} 32 \quad$ Divide.
8 is not a solution of the equation $\frac{y}{4}=32$.
C. $8 x=72 ; x=9$

8(9) $\stackrel{?}{=} 72 \quad$ Substitute 9 for $x$.
$72 \stackrel{?}{=} 72 \quad$ Multiply.

## Math Talk

Mathematical Practices
How would you state each equation in the Example in words?

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## YOUR TURN

Determine whether the given value is a solution of the equation.

1. $11=n+6 ; n=5$
2. $y-6=24 ; y=18$
3. $\frac{x}{9}=4 ; x=36$
4. $15 t=100 ; t=6$

## Writing Equations to Represent Situations

You can represent some real-world situations with an equation. Making a model first can help you organize the information.

## EXAMPLE 2 <br>  <br> Mark scored 17 points for the home team in a basketball game. His teammates as a group

 scored $p$ points. Write an equation to represent this situation.


HOME Peniod GUEST

7. Craig is c years old. His 12 -year-old sister Kaitlin is 3 years younger than Craig.
6. Juanita has 102 beads to make $n$ necklaces. Each necklace will have 17 beads.
$\qquad$
8. Kim rented skates for $h$ hours. The rental fee was $\$ 2$ per hour and she paid a total of $\$ 8$.

## Writing an Equation and Checking Solutions

You can substitute a given value for the variable in a real-world equation to

## EXAMPLE 3



COMMON CORE

## 6.EE. 7

Sarah used a gift card to buy $\mathbf{\$ 4 7}$ worth of groceries. Now she has $\mathbf{\$ 1 8}$ left on her gift card. Write an equation to represent this situation. Use your equation to determine whether Sarah had \$65 or \$59 on the gift card before buying groceries.

STEP 1 Write a word equation based on the situation.

STEP 2 Rewrite the equation using a variable for the unknown quantity and the given values for the known quantities.

Let $x$ be the amount on the card.

| Amount <br> on card | - | Amount <br> spent | $=$Amount left <br> on card |  |
| :---: | :---: | :---: | :---: | :---: |
| $x$ | - | 47 | $=$ | 18 |

STEP 3 Substitute 65 and 59 for $x$ to see which equation is true.

The amount spent and the amount left on the card are the known quantities. Substitute those values in the equation.

$$
\begin{array}{rrr}
x-47=18 & x-47 & =18 \\
65-47 & \stackrel{?}{=} 18 & 59-47 \\
18 & \stackrel{?}{=} 18 \\
= & 12 & \stackrel{?}{=} 18
\end{array}
$$

- The amount on Sarah's gift card before she bought groceries was $\$ 65$.


## Reflect

9. What If? Suppose Sarah has $\$ 12$ left on her gift card. How would this change the equation and the final answer?
$\qquad$
$\qquad$

## YOUR TURN

10. On Saturday morning, Owen earned $\$ 24$. By the end of the afternoon he had earned a total of $\$ 62$. Write an equation represent the situation. Determine whether Owen earned $\$ 38$ or $\$ 31$ on Saturday afternoon.

## Guided Practice

## Determine whether the given value is a solution of the equation. (Example 1)

1. $23=x-9 ; x=14$


2. $14+x=46 ; x=32$ $\qquad$
3. $25=\frac{k}{5} ; k=5$ $\qquad$
4. $21=m+9 ; m=11$ $\qquad$
5. $d-4=19 ; d=15$ $\qquad$
6. $w-9=0 ; w=9$ $\qquad$
7. Each floor of a hotel has $r$ rooms. On 8 floors, there are a total of 256 rooms.

Write an equation to represent this situation. (Example 2)

| Number |
| :--- |

14. In the school band, there are 5 trumpet players and $f$ flute players. There are twice as many flute players as there are trumpet players. Write an equation to represent this situation. (Example 2)
15. Pedro bought 8 tickets to a basketball game. He paid a total of $\$ 208$. Write an equation to determine whether each ticket cost $\$ 26$ or $\$ 28$. (Example 3)
$\qquad$
16. The high temperature was $92^{\circ} \mathrm{F}$. This was $24^{\circ} \mathrm{F}$ higher than the overnight low temperature. Write an equation to determine whether the low temperature was $62^{\circ} \mathrm{F}$ or $68^{\circ} \mathrm{F}$. (Example 3)

## ESSENTIAL QUESTION CHECK-IN

17. Tell how you can determine whether a number is a solution of an equation.

### 11.1 Independent Practice

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18. Andy is one-fourth as old as his grandfather, who is 76 years old. Write an equation to determine whether Andy is 19 or 22 years old.
19. A sleeping bag weighs 8 pounds. Your backpack and sleeping bag together weigh 31 pounds. Write an equation to determine whether the backpack without the sleeping bag weighs 25 or 23 pounds.
20. Halfway through a bus route, 23 students have been dropped off and 48 students remain on the bus. Write an equation to determine whether there are 61 or 71 students on the bus at the beginning of the route.
$\qquad$
21. Write an equation that involves multiplication, contains a variable, and has a solution of 5 . Then write another equation that has the same solution and includes the same variable and numbers but uses division.
$\qquad$
$\qquad$
22. Vocabulary How are expressions and equations different?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$五
23. Multistep Alan has partially completed a table showing the distances between his town, Greenville, and two other towns.

| Distance between Greenville and <br> Nearby Towns (miles) |  |
| :--- | :---: |
| Parker | 29 |
| Hadley | $?$ |

a. The distance between Hadley and Greenville is 13 miles less than the distance between Parker and Greenville. Write two equations that compare the distance between Hadley and Greenville and the distance between Parker and Greenville. Tell what your variable represents.
$\qquad$
$\qquad$
$\qquad$
b. Alan says the distance from Hadley to Greenville is 16 miles. Is he correct? Explain.
24. Explain the Error A problem states that Ursula earns $\$ 9$ per hour. To write an expression that tells how much money Ursula earns for $h$ hours, Joshua wrote $\frac{9}{h}$. Sarah wrote 9h. Whose expression is correct and why?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
25. Communicate Mathematical Ideas A dog weighs 44 pounds and the veterinarian thinks it needs to lose 7 pounds. Mikala wrote the equation $x+7=44$ to represent the situation. Kirk wrote the equation $44-x=7$. Which equation is correct? Can you write another equation that represents the situation?
$\qquad$
$\qquad$
26. Multiple Representations The table shows the ages of Cindy and her dad.
a. Write an equation that relates Cindy's age to her dad's age when Cindy is 18 . Tell what the variable

| Dad's Age | Cindy's Age |
| :---: | :---: |
| 28 years old | 2 years old |
| 36 years old | 10 years old |
| $?$ | 18 years old | represents.

b. Determine if 42 is a solution to the equation. Show your work.
c. Explain the meaning of your answer in part $\mathbf{b}$.

## H.0.7.

FOCUS ON HIGHER ORDER THINKING
27. Critical Thinking In the school band, there are 4 trumpet players and $f$ flute players. The total number of trumpet and flute players is 12 . Are there twice as many flute players as trumpet players? Explain.
$\qquad$
$\qquad$
28. Problem Solving Ronald paid $\$ 162$ for 6 tickets to a basketball game. During the game he noticed that his friend paid $\$ 130$ for 5 tickets. The price of each ticket was $\$ 26$. Was Ronald overcharged? Explain.
$\qquad$
$\qquad$
29. Communicate Mathematical Ideas Tariq said you can write an equation by setting an expression equal to itself. Would an equation like this be true? Explain.
$\qquad$
$\qquad$
$\qquad$

How do you solve equations that contain addition or subtraction?

## EXPLORE ACTIVITY Rearc

COMMON
CORE
6.EE.6, 6.EE. 7

## Modeling Equations

A puppy weighed 6 ounces at birth. After two weeks, the puppy weighed 14 ounces. How much weight did the puppy gain?

Let $x$ represent the number of ounces gained.


To answer this question, you can solve the equation $6+x=14$.
Algebra tiles can model some equations. An equation mat represents the two sides of an equation. To solve the equation, remove the same number of tiles from both sides of the mat until the $x$ tile is by itself on one side.
(A) Model $6+x=14$.

B How many 1 tiles must you remove on the left side so that the $x$ tile is by itself? $\qquad$ Cross out these tiles on the equation mat.


C Whenever you remove tiles from one side of the mat, you must remove the same number of tiles from the other side of the mat. Cross out the tiles that should be removed on the right side of the mat.
D How many tiles remain on the right side of the mat? $\qquad$ This is the solution of the equation.

The puppy gained $\qquad$ ounces.

## Reflect

1. Communicate Mathematical Ideas How do you know when the model shows the final solution? How do you read the solution?


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## Using Subtraction to Solve Equations

Removing the same number of tiles from each side of an equation mat models subtracting the same number from both sides of an equation.

## Subtraction Property of Equality

You can subtract the same number from both sides of an equation, and the two sides will remain equal.

When an equation contains addition, solve by subtracting the same number from both sides.

## EXAMPLE 1

COMMON
CORE

## 6.EE. 5

Solve the equation $a+15=26$. Graph the solution on a number line.
$a+15=26 \quad$ Notice that the number 15 is added to a.
$a+15=26$
$\frac{-15-15}{a-11}$ Subtract 15 from both sides of the equation.
Check: $a+15=26$

$$
\begin{aligned}
11+15 & \stackrel{?}{=} 26
\end{aligned} \text { Substitute } 11 \text { for a. }
$$

Graph the solution on a number line.


## Reflect

2. Communicate Mathematical Ideas How do you decide which number to subtract from both sides?
$\qquad$
$\qquad$

YOUR TURN
3. Solve the equation $5=w+1.5$.


Graph the solution on a number line.
$w=$ $\qquad$

## Using Addition to Solve Equations

When an equation contains subtraction, solve by adding the same number to both sides.

## Addition Property of Equality

You can add the same number to both sides of an equation, and the two sides will remain equal.

## EXAMPLE 2

Solve the equation $y-21=18$. Graph the solution on a number line.

```
y-21=18 Notice that the number 21 is subtracted from y.
y-21= 18
    +21+21 Add 21 to both sides of the equation.
Check: y - 21=18
\[
\begin{array}{rlr}
39-21 & \stackrel{?}{=} 18 & \text { Substitute } 39 \text { for } y . \\
18 & \stackrel{?}{=} 18 & \text { Subtract. }
\end{array}
\]
```

Graph the solution on a number line.


## Reflect

4. Communicate Mathematical Ideas How do you know whether to add on both sides or subtract on both sides when solving an equation?
$\qquad$
$\qquad$
$\qquad$

## YOUR TURN

5. Solve the equation $h-\frac{1}{2}=\frac{3}{4}$.
 Graph the solution on a number line.
$h=$ $\qquad$

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## Solving Equations that Represent Geometric Concepts

You can write equations to represent geometric relationships.
Recall that a straight line has an angle measure of $180^{\circ}$. Two angles whose measures have a sum of $180^{\circ}$ are called supplementary angles. Two angles whose measures have a sum of $90^{\circ}$ are called complementary angles.

## EXAMPLE 3

## COMMON <br> CORE <br> 6.EE.7, 6.EE. 6

Find the measure of the unknown angle.
STEP 1 Write the information in the boxes.


STEP 2 Write a description to represent the model. Include a question for the unknown angle.

The sum of an unknown angle and a $60^{\circ}$ angle is $180^{\circ}$. What is the measure of the unknown angle?

STEP 3 Write an equation.
$x+60^{\circ}=180^{\circ}$
STEP 4 Solve the equation.
$x+60^{\circ}=180^{\circ}$


Subtract 60 $x \quad=120^{\circ} \quad$ from each side.
$\therefore \quad$ The unknown angle measures $120^{\circ}$.


## YOUR TURN

6. Write and solve an equation to find the measure of the unknown angle.
$\qquad$

## Writing Real-World Problems for a Given Equation

## EXAMPLE 4

Red
world
COMMON CORE

## 6.EE. 7

Write a real-world problem for the equation $21.79+x=25$. Then solve the equation.
$21.79+x=25$
STEP 1 Examine each part of the equation.
$x$ is the unknown or quantity we are looking for.
21.79 is added to $x$.
$=25$ means that after adding 21.79 and $x$, the result is 25 .


## Math Talk

Mathematical Practices

STEP 2 Write a real-world situation that involves adding two quantities.
Joshua wants to buy his mother flowers and a card for Mother's Day. Joshua has $\$ 25$ to spend and selects roses for $\$ 21.79$. How much can he spend on a card?

STEP 3 Solve the equation.

| $21.79+x$ | $=$ |
| ---: | :--- |
| -25 |  |
| -21.79 |  |
| $x$ | $=\frac{-21.79}{3.21}$ |

The final answer includes units of money in dollars.

How is the question in a real-world problem related to its equation?

- Joshua can spend $\$ 3.21$ on a Mother's Day card.


## Reflect

8. What If? How might the real-world problem change if the equation was $x-21.79=25$ and you still used roses for $\$ 21.79$ ?
$\qquad$
$\qquad$
$\qquad$

## YOUR TURN

9. Write a real-world problem for the equation $x-100=40$. Then solve the equation.

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## Guided Practice

1. A total of 14 guests attended a birthday party. Three friends stayed after the party to help clean up. How many left when the party ended? (Explore Activity)
a. Let $x$ represent the $\qquad$
b.

c. Draw algebra tiles to model the equation.
$\qquad$ friends left when the party ended. $\square$
Solve each equation. Graph the solution on a number
line. (Examples 1 and 2)
2. $2=x-3$
$x=$ $\qquad$

3. $s+12.5=14$
$s=$ $\qquad$


Solve each equation. (Examples 1 and 2)
4. $h+6.9=11.4$
$h=$ $\qquad$
5. $82+p=122$
$p=$ $\qquad$
6. $n+\frac{1}{2}=\frac{7}{4}$
$n=$ $\qquad$
7. Write and solve an equation to find the measure of the unknown angle. (Example 3)

8. Write a real-world problem for the equation $x-75=200$.

Then solve the equation. (Example 4)

## ESSENTIAL QUESTION CHECK-IN

9. How do you solve equations that contain addition or subtraction?

### 11.2 Independent Practice

## Write and solve an equation to answer each question.

10. A wildlife reserve had 8 elephant calves born during the summer and now has 31 total elephants. How many elephants were in the reserve before summer began?
$\qquad$
$\qquad$
11. My sister is 14 years old. My brother says that his age minus twelve is equal to my sister's age. How old is my brother?
$\qquad$
12. Kim bought a poster that cost $\$ 8.95$ and some colored pencils. The total cost was $\$ 21.35$. How much did the colored pencils cost?
$\qquad$
$\qquad$
13. The Acme Car Company sold 37 vehicles in June. How many compact cars were sold in June?

| Acme Car Company - June Sales |  |
| :---: | :---: |
| Type of car | Number sold |
| SUV | 8 |
| Compact | $?$ |

14. Sandra wants to buy a new MP3 player that is on sale for $\$ 95$. She has saved $\$ 73$. How much more money does she need?
$\qquad$
$\qquad$
15. Ronald spent $\$ 123.45$ on school clothes. He counted his money and discovered that he had $\$ 36.55$ left. How much money did he originally have?
$\qquad$
$\qquad$
16. Brita withdrew $\$ 225$ from her bank account. After her withdrawal, there was \$548 left in Brita's account. How much money did Brita have in her account before the withdrawal?
17. Represent Real-World Problems Write a real-world situation that can be represented by $15+c=17.50$. Then solve the equation and describe what your answer represents for the problem situation.
$\qquad$
$\qquad$
$\qquad$
18. Critique Reasoning Paula solved the equation $7+x=10$ and got 17 , but she is not certain if she got the correct answer. How could you explain Paula's mistake to her?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
19. Multistep A grocery store is having a sale this week. If you buy a 5-pound bag of apples for the regular price, you can get another bag for $\$ 1.49$. If you buy a 5 -pound bag of oranges at the regular price, you can get another bag for $\$ 2.49$.

| Grocery Prices |  |
| :---: | :---: |
|  | Regular price |
| 5-pound bag of apples | $\$ 2.99$ |
| 5-pound bag of oranges | $\$ 3.99$ |

a. Write an equation to find the discount for each situation using $a$ for the amount of the discount for apples and $r$ for the amount of the discount for oranges.
$\qquad$
b. Which fruit has a greater discount? Explain.
20. Critical Thinking An orchestra has twice as many woodwind instruments as brass instruments. There are a total of 150 brass and woodwind instruments.
a. Write two different addition equations that describe this situation. Use $w$ for woodwinds and $b$ for brass.
b. How many woodwinds and how many brass instruments satisfy the given information?
21. Look for a Pattern Assume the following: $a+1=2, b+10=20$, $c+100=200, d+1,000=2,000, \ldots$
a. Solve each equation for each variable.
b. What pattern do you notice between the variables?
c. What would be the value of $g$ if the pattern continues?

## EXPLORE ACTIVITY

COMMON CORE
6.EE. 6

## Modeling Equations

Deanna has a cookie recipe that requires 12 eggs to make 3 batches of cookies. How many eggs are needed per batch of cookies?

Let $x$ represent the number of eggs needed per batch.


To answer this question, you can use algebra tiles to solve $3 x=12$.
A
Model $3 x=12$.


B There are $3 x$ tiles, so draw circles to separate the tiles into 3 equal groups. One group has been circled for you.


C How many 1 tiles are in each group? $\qquad$ This is the solution of the equation.
$\qquad$ eggs are needed per batch of cookies.

## Reflect

1. Look for a Pattern Why does it make sense to arrange the twelve 1 tiles in 3 rows of 4 instead of any other arrangement of twelve 1 tiles, such as 2 rows of 6?

## Using Division to Solve Equations

Separating the tiles on both sides of an equation mat into an equal number of groups models dividing both sides of an equation by the same number.

## Division Property of Equality

You can divide both sides of an equation by the same nonzero number, and the two sides will remain equal.

When an equation contains multiplication, solve by dividing both sides of the equation by the same nonzero number.

## EXAMPLE 1

## COMMON

CORE

## Solve each equation. Graph the solution on a number line.

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A $9 a=54$

$$
\begin{aligned}
9 a=54 & \text { Notice that } 9 \text { is multiplied by } a . \\
\frac{9 a}{9}=\frac{54}{9} & \text { Divide both sides of the equation by } 9 . \\
a=6 & \leftarrow
\end{aligned}
$$

Check: $9 a=54$

9(6) $\stackrel{?}{=} 54$
$54 \stackrel{?}{=} 54$
Substitute 6 for a.
Multiply on the left side.

B $\mathbf{1 8}=\mathbf{6 d}$
$18=6 d \quad$ Notice that 6 is multiplied by $d$.
$\frac{18}{6}=\frac{6 d}{6} \quad$ Divide both sides of the equation by 6.
$3=d$


Check: $18=6 d$

| $18 \stackrel{?}{=} 6(3)$ | Substitute 3 for $d$. |
| :--- | :--- |
| $18 \stackrel{?}{=} 18$ | Multiply on the right side. |

## YOUR TURN

Solve the equation $3 x=21$. Graph the solution on a number line.
$\qquad$
2. $x=$


## Using Multiplication to Solve Equations

When an equation contains division, solve by multiplying both sides of the equation by the same number.

## Multiplication Property of Equality

You can multiply both sides of an equation by the same number, and the two sides will remain equal.

## EXAMPLE 2

## 6.EE. 5

Solve each equation. Graph the solution on a number line.
(A) $\frac{x}{5}=10$
$\frac{x}{5}=10 \quad$ Notice that $x$ is divided by the number 5 .
$5 \cdot \frac{x}{5}=5 \cdot 10$
$x=50$
Check: $\frac{x}{5}=10$

$$
\begin{array}{ll}
\frac{50}{5} \stackrel{?}{=} 10 & \text { Substitute } 50 \text { for } x \\
10 \stackrel{?}{=} 10 & \text { Divide on the left side. }
\end{array}
$$

(B) $15=\frac{r}{2}$
$15=\frac{r}{2} \quad$ Notice that $r$ is divided by the number 2.
$2 \cdot 15=2 \cdot \frac{r}{2} \quad$ Multiply both sides of the equation by 2.
$30=r$
Check: $15=\frac{r}{2}$

$$
\begin{array}{ll}
15 \stackrel{?}{=} \frac{30}{2} & \text { Substitute } 30 \text { for } r \\
15 \stackrel{?}{=} 15 & \text { Divide on the right side }
\end{array}
$$

## Math Talk

Mathematical Practices
How is solving a multiplication equation similar to solving a division equation? How is it different?


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My Notes

## Using Equations to Solve Problems

You can use equations to solve real-world problems.

## EXAMPLE 3 <br> problem Solving COMMON CORE <br> 6.EE. 7

## Juanita is scrapbooking. She usually completes about 9 pages per hour. One night last week she completed pages 23 through 47 in 2.5 hours. Did she work at her average rate?

## Analyze Information



Identify the important information.

- Worked for 2.5 hours
- Starting page: 23 Ending page: 47
- Scrapbooking rate: 9 pages per hour


## Formulate a Plan

- Solve an equation to find the number of pages Juanita can expect to complete.
- Compare the number of pages Juanita can expect to complete with the number of pages she actually completed.


## Solve

Let $n$ represent the number of pages Juanita can expect to complete in 2.5 hours if she works at her average rate of 9 pages per hour.

Write an equation.

$$
\begin{aligned}
\frac{n}{2.5} & =9 & & \text { Write the equation. } \\
2.5 \cdot \frac{n}{2.5} & =2.5 \cdot 9 & & \text { Multiply both sides by } 2.5 \\
n & =22.5 & &
\end{aligned}
$$

Juanita can expect to complete 22.5 pages in 2.5 hours.
Juanita completed pages 23 through 47, a total of 25 pages. Because $25>22.5$, she worked faster than her expected rate.

## Justify and Evaluate

You used an equation to find the number of pages Juanita could expect to complete in 2.5 hours if she worked at her average rate. You found that she could complete 22.5 pages.

Since 22.5 pages is less than the 25 pages Juanita completed, she worked faster than her average rate.

The answer makes sense, because Juanita completed 25 pages in 2.5 hours, which is equivalent to a rate of 10 pages in 1 hour.

Since $10>9$, you know that she worked faster than her average rate.
4. Roberto is dividing his baseball cards equally among himself, his brother, and his 3 friends. Roberto was left with 9 cards. How many cards did Roberto give away? Write and solve an equation to solve the problem.

## Writing Real-World Problems

You can write a real-world problem for a given equation.

Write a real-world problem for the equation $8 x=72$. Then solve the problem.


STEP 1 Examine each part of the equation.
$x$ is the unknown value you want to find.
8 is multiplied by $x$.
$=72$ means that after multiplying 8 and $x$, the result is 72 .
STEP 2 Write a real-world situation that involves multiplying two quantities.
A hot air balloon flew at 8 miles per hour. Write and solve a multiplication equation to find out how many hours the balloon traveled if it covered a distance of 72 miles.

STEP 3 Use the equation to solve the problem.
$8 x=72$
$\frac{8 x}{8}=\frac{72}{8} \quad$ Divide both sides by 8.
$x=9$

- The balloon traveled for 9 hours.


## YOUR TURN

5. Write a real-world problem for the equation $11 x=385$. Then solve the problem.


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## Guided Practice

1. Caroline ran 15 miles in 5 days. She ran the same distance each day. Write and solve an equation to determine the number of miles she ran each day. (Explore Activity)
a. Let $x$ represent the $\qquad$ -
b. Number of
 $=\square$
$\qquad$ $=$ $\qquad$
c. Draw algebra tiles to model the equation. Caroline ran $\qquad$ miles each day.


Solve each equation. Graph the solution on a number line.
(Examples 1 and 2)
2. $x \div 3=3 ; x=$ $\qquad$

3. $4 x=32 ; x=$ $\qquad$

4. The area of the rectangle shown is 24 square inches. How much longer is its length than its width? (Example 3)
$\qquad$
$\qquad$
5. Write a real-world problem for the equation $15 w=45$. Then solve the problem. (Example 4)
$\qquad$
$\qquad$

ESSENTIAL QUESTION CHECK-IN
6. How do you solve equations that contain multiplication or division?

### 11.3 Independent Practice


12. Dharmesh has a square garden with a perimeter of 132 feet. Is the area of the garden greater than 1,000 square feet?

$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
13. Ingrid walked her dog and washed her car. The time she spent walking her dog was one-fourth the time it took her to wash her car. It took Ingrid 14 minutes to walk the dog. How long did it take Ingrid to wash her car?
$\qquad$
14. Representing Real-World Problems Write and solve a problem involving money that can be solved with a multiplication equation.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
15. Representing Real-World Problems Write and solve a problem involving money that can be solved with a division equation and has a solution of 1,350.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
16. Communicating Mathematical Ideas Explain why $7 \cdot \frac{x}{7}=x$. How does your answer help you solve a division equation such as $\frac{x}{7}=2$ ?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
17. Critical Thinking A number tripled and tripled again is 729 . What is the number? Show your work.
$\qquad$
$\qquad$
18. Multistep Andre has 4 times as many model cars as Peter, and Peter has one-third as many model cars as Jade. Andre has 36 model cars.
a. Write and solve an equation to find how many model cars Peter has.
$\qquad$
b. Using your answer from part a, write and solve an equation to find how many model cars Jade has.
19. Persevere in Problem Solving The area of a rectangle is 42 square inches and one side is 12 inches long. Find the perimeter of the rectangle. Show your work.

## EXPLORE ACTIVITY

## Using Inequalities to Describe Quantities

You can use inequality symbols with variables to describe quantities that can have many values.

| Symbol | Meaning | Word Phrases |
| :---: | :---: | :---: |
| $<$ | Is less than | Fewer than, below |
| $>$ | Is greater than | More than, above |
| $\leq$ | Is less than or equal to | At most, no more than |
| $\geq$ | Is greater than or equal to | At least, no less than |

A The lowest temperature ever recorded in Florida was $-2^{\circ} \mathrm{F}$. Graph this temperature on the number line.

(B The temperatures $0^{\circ} \mathrm{F}, 3^{\circ} \mathrm{F}, 6^{\circ} \mathrm{F}, 5^{\circ} \mathrm{F}$, and $-1^{\circ} \mathrm{F}$ have also been recorded in Florida. Graph these temperatures on the number line.

C How do the temperatures in B compare to -2? How can you see this relationship on the number line?

D How many other numbers have the same relationship to -2 as the temperatures in B ? Give some examples.

E Suppose you could graph all of the possible answers to D on a number line. What would the graph look like?

F Let $x$ represent all the possible answers to $\mathbf{D}$.
Complete this inequality: $x$ $\square$ $-2$

## Graphing the Solutions of an Inequality

A solution of an inequality that contains a variable is any value of the variable that makes the inequality true. For example, 7 is a solution of $x>-2$, since $7>-2$ is a true statement.

## EXAMPLE 1

COMMON CORE

## Graph the solutions of each inequality. Check the solutions.

A $y \leq-3$
STEP 1 Draw a solid circle at -3 to show that -3 is a solution.
STEP 2 Shade the number line to the left of -3 to show that numbers less than -3 are solutions.

Use a solid circle for an inequality that uses $\geq$ or $\leq$.


STEP 3 Check your solution.
Choose a number that is on the shaded section of the number line, such as -4 . Substitute -4 for $y$.

- $\quad-4 \leq-3 \quad-4$ is less than -3 , so -4 is a solution.

B $1<m$
STEP 1 Draw an empty circle at 1 to show that 1 is not a solution.

STEP 2 Shade the number line to the right of 1 to show that numbers greater than 1 are solutions.


STEP 3 Check your answer.
Substitute 2 for $m$.

- $\quad 1<2 \quad 1$ is less than 2 , so 2 is a solution.


## Reflect

1. Critique Reasoning Inez says you can rewrite $1<m$ as $m>1$. Do you agree?

## YOUR TURN

3. Graph the solution of the inequality $t \leq-4$.


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## Writing Inequalities

You can write an inequality to model the relationship between an algebraic expression and a number. You can also write inequalities to represent certain real-world situations.

## EXAMPLE 2



COMMON CORE

## 6.EE.8, 6.EE. 6

A Write an inequality that represents the phrase the sum of $y$ and 2 is greater than 5. Draw a graph to represent the inequality.

STEP 1 Write the inequality.
The sum of $y$ and 2 is greater than 5 .

$$
y+2>5
$$

STEP 2 Graph the solution.
For $y+2$ to have a value greater than 5 , $y$ must be a number greater than 3 .


STEP 3 Check your solution by substituting a number greater than 3, such as 4 , into the original inequality.
$4+2>5$ Substitute 4 for $y$.
$6>5 \quad 6$ is greater than 5 , so 4 is a solution.
B To test the temperature rating of a coat, a scientist keeps the temperature below $5^{\circ} \mathrm{C}$. Write and graph an inequality to represent this situation.

STEP 1 Write the inequality. Let $t$ represent the temperature in the lab.

$$
t<5 \quad \text { The temperature must be less than } 5^{\circ} \mathrm{C} .
$$

STEP 2 Graph the inequality.
o


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## YOUR TURN

4. Write an inequality that represents the phrase the sum of 1 and y is greater than or equal to 3 . Check to see if $y=1$ is a solution.
$\qquad$
$\qquad$

## Write and graph an inequality to represent each situation.

5. The temperature in February was at most $6^{\circ} \mathrm{F}$. $\qquad$

6. Each package must weigh more than 2 ounces.


## Guided Practice

1. Graph $1 \leq x$. Use the graph to determine which of these numbers are solutions of the inequality: $-1,3,0,1$ (Explore Activity and Example 1)
$\qquad$

2. Graph $-3>z$. Check the graph using substitution. (Example 1)
3. Write an inequality that represents the phrase "the sum of 4 and $x$ is greater than 6." Draw a graph that represents
 the inequality, and check your solution. (Example 2)
$\qquad$
4. During hibernation, a garter snake's body temperature never goes below $3^{\circ} \mathrm{C}$. Write and graph an inequality
 that represents this situation. (Example 2)

## ESSENTIAL QUESTION CHECK-IN

5. Write an inequality to represent this situation: Nina wants to take at least $\$ 15$ to the movies. How did you decide which inequality symbol to use?

### 11.4 Independent Practice

## COMMON <br> CORE

6. Which of the following numbers are solutions to $x \geq 0$ ?

$$
-5,0.03,-1,0,1.5,-6, \frac{1}{2}
$$

## Graph each inequality.

7. $t \leq 8$

8. $-7<h$

9. $x \geq-9$

10. $n>2.5$

11. $-4 \frac{1}{2}>x$


Write an inequality that matches the number line model.
12. $\qquad$

13. $\qquad$

14. $\qquad$

15. $\qquad$

16. A child must be at least 48 inches tall to ride a roller coaster.
a. Write and graph an inequality to represent this situation.

$\qquad$
b. Can a child who is 46 inches tall ride the roller coaster? Explain.

## Write and graph an inequality to represent each situation.

17. The stock is worth at least $\$ 14.50$. $\qquad$

18. The temperature is less than $3.5^{\circ} \mathrm{F}$. $\qquad$

19. The goal of the fundraiser is to make more than $\$ 150$.

focus on higher order thinking
20. Communicate Mathematical Ideas Explain how to graph the inequality $8 \geq y$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
21. Represent Real-World Problems The number line shows an inequality. Describe a real-world situation that the inequality could represent.

22. Critique Reasoning Natasha is trying to represent the following situation with a number line model: There are fewer than 5 students in the cafeteria. She has come up with two possible representations, shown below. Which is the better representation, and why?


## Ready to Go On?

### 11.1 Writing Equations to Represent Situations

Determine whether the given value is a solution of the equation.

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1. $\frac{b}{12}=5 ; b=60$ $\qquad$ 2. $7 w=87 ; w=12$

## Write an equation to represent the situation.

3. The number of eggs in the refrigerator $e$ decreased by 5 equals 18 .

### 11.2 Addition and Subtraction Equations

Solve each equation.
4. $r-38=9$ $\qquad$ 5. $h+17=40$
$\qquad$
6. $n+75=155$ $\qquad$ 7. $q-17=18$ $\qquad$

### 11.3 Multiplication and Division Equations

## Solve each equation.

8. $8 z=112$ $\qquad$ 9. $\frac{d}{14}=7$ $\qquad$
9. $\frac{f}{28}=24$ $\qquad$ 11. $3 a=57$ $\qquad$

### 11.4 Writing Inequalities

Write an inequality to represent each situation, then graph the solutions.
12. There are fewer than 8 gallons of gas in the tank. $\qquad$

13. There are at least 3 slices of bread left in the bag. $\qquad$


## ESSENTIAL QUESTION

14. How can you solve problems involving equations that contain addition, subtraction, multiplication, or division?

## Selected Response

1. Kate has gone up to the chalkboard to do math problems 5 more times than Andre. Kate has gone up 11 times. Which equation represents this situation?
(A) $a-11=5$
(B) $5 a=11$
(C) $a-5=11$
(D) $a+5=11$
2. For which equation is $y=7$ a solution?
(A) $7 y=1$
(B) $18=11+y$
(C) $y+7=0$
(D) $\frac{y}{2}=14$
3. Which is an equation?
(A) $17+x$
(C) $20 x=200$
(B) $45 \div x$
(D) $90-x$
4. The temperature never rose above $6^{\circ} \mathrm{F}$ on Friday. Which number line could represent this situation?
(A)

(B)

(C)

(D)

5. Becca hit 7 more home runs than Beverly. Becca hit 21 home runs. How many home runs did Beverly hit?
(A) 3
(C) 21
(B) 14
(D) 28
6. Jeordie spreads out a rectangular picnic blanket with an area of 42 square feet. Its width is 6 feet. Which equation could you use to find its length?
(A) $6 x=42$
(C) $\frac{6}{x}=42$
(B) $42-x=6$
(D) $6+x=42$
7. What is a solution to the equation $6 t=114$ ?
(A) $t=19$
(C) $t=120$
(B) $t=108$
(D) $t=684$
8. The area of a rectangular deck is 680 square feet. The deck's width is 17 feet. What is its perimeter?
(A) 40 feet
(C) 114 feet
(B) 57 feet
(D) 228 feet

## Mini-Task

9. Sylvia earns $\$ 7$ per hour at her afterschool job. After working one week, she received a paycheck for $\$ 91$.
a. Write and solve an equation to find the number of hours Sylvia worked to earn $\$ 91$.
b. The greatest number of hours Sylvia can work in any week is 15 . Write an inequality to represent this statement.
c. What is the greatest amount of money Sylvia can earn in one week?
