

Relationships in Two Variables

MODULE



12



ESSENTIAL QUESTION

How can you use relationships in two variables to solve real-world problems?

LESSON 12.1

Graphing on the Coordinate Plane

COMMON CORE 6.NS.6, 6.NS.6b

LESSON 12.2

Independent and Dependent Variables in Tables and Graphs

COMMON CORE 6.EE.9

LESSON 12.3

Writing Equations from Tables

COMMON CORE 6.EE.9

LESSON 12.4

Representing Algebraic Relationships in Tables and Graphs

COMMON CORE 6.EE.9



Real-World Video

A two-variable equation can represent an animal's distance over time. A graph can display the relationship between the variables. You can graph two or more animals' data to visually compare them.

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Are YOU Ready?

Complete these exercises to review skills you will need for this module.



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Multiplication Facts

EXAMPLE $8 \times 7 = \square$

Use a related fact you know.

$$7 \times 7 = 49$$

$$\begin{aligned} \text{Think: } 8 \times 7 &= (7 \times 7) + 7 \\ &= 49 + 7 \\ &= 56 \end{aligned}$$

Multiply.

1. 7×6 _____ 2. 10×9 _____ 3. 13×12 _____ 4. 8×9 _____

Write the rule for each table.

5.

x	1	2	3	4
y	7	14	21	28

6.

x	1	2	3	4
y	7	8	9	10

7.

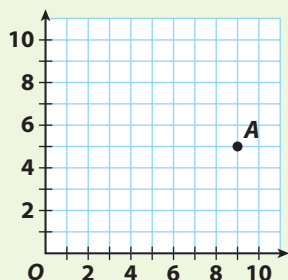
x	1	2	3	4
y	5	10	15	20

8.

x	0	4	8	12
y	0	2	4	6

Graph Ordered Pairs (First Quadrant)

EXAMPLE



Start at the origin.
Move 9 units right.
Then move 5 units up.
Graph point A(9, 5).

Graph each point on the coordinate grid above.

9. B(0, 8) 10. C(2, 3) 11. D(6, 7) 12. E(5, 0)

Reading Start-Up

Visualize Vocabulary

Use the ✓ words to complete the chart.

Parts of the Algebraic Expression $14 + 3x$		
Definition	Mathematical Representation	Review Word
A specific number whose value does not change	14	
A number that is multiplied by a variable in an algebraic expression	3	
A letter or symbol used to represent an unknown	x	

Understand Vocabulary

Complete the sentences using the preview words.

1. The numbers in an ordered pair are _____.
2. A _____ is formed by two number lines that intersect at right angles.

Vocabulary

Review Words

- ✓ coefficient (*coeficiente*)
- ✓ constant (*constante*)
- equation (*ecuación*)
- negative number (*número negativo*)
- positive number (*número positivo*)
- scale (*escala*)
- ✓ variable (*variable*)

Preview Words

- axes (*ejes*)
- coordinate plane (*plano cartesiano*)
- coordinates (*coordenadas*)
- dependent variable (*variable dependiente*)
- independent variable (*variable independiente*)
- ordered pair (*par ordenado*)
- origin (*origen*)
- quadrants (*cuadrante*)
- x-axis (*eje x*)
- x-coordinate (*coordenada x*)
- y-axis (*eje y*)
- y-coordinate (*coordenada y*)

Active Reading

Layered Book Before beginning the module, create a layered book to help you learn the concepts in this module. Label each flap with lesson titles from this module. As you study each lesson, write important ideas such as vocabulary and formulas under the appropriate flap. Refer to your finished layered book as you work on exercises from this module.





Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

COMMON CORE

6.EE.9

Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. ...

Key Vocabulary

equation (*ecuación*)

A mathematical sentence that shows that two expressions are equivalent.

What It Means to You

You will learn to write an equation that represents the relationship in a table.

UNPACKING EXAMPLE 6.EE.9

Emily has a dog-walking service. She charges a daily fee of \$7 to walk a dog twice a day. Create a table that shows how much Emily earns for walking 1, 6, 10, and 15 dogs. Write an equation that represents the situation.

Dogs walked	1	6	10	15
Earnings (\$)	7	42	70	105

Earnings is 7 times the number of dogs walked. Let the variable e represent earnings and the variable d represent the number of dogs walked.

$$e = 7 \times d$$

COMMON CORE

6.EE.9

...Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

Key Vocabulary

coordinate plane

(*plano cartesiano*)

A plane formed by the intersection of a horizontal number line called the x -axis and a vertical number line called the y -axis.

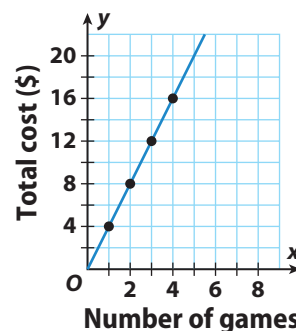
What It Means to You

You can use words, a table, a graph, or an equation to model the same mathematical relationship.

UNPACKING EXAMPLE 6.EE.9

The equation $y = 4x$ represents the total cost y for x games of miniature golf. Make a table of values and a graph for this situation.

Number of games, x	1	2	3	4
Total cost (\$), y	4	8	12	16



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LESSON 12.1 Graphing on the Coordinate Plane

COMMON CORE 6.NS.6c

... find and position pairs of integers and other rational numbers on a coordinate plane. Also 6.NS.6, 6.NS.6b, 6.NS.8



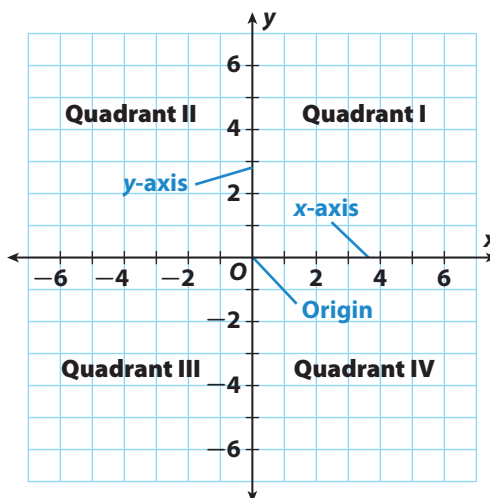
ESSENTIAL QUESTION

How do you locate and name points in the coordinate plane?

Naming Points in the Coordinate Plane

A **coordinate plane** is formed by two number lines that intersect at right angles. The point of intersection is 0 on each number line.

- The two number lines are called the **axes**.
- The horizontal axis is called the **x-axis**.
- The vertical axis is called the **y-axis**.
- The point where the axes intersect is called the **origin**.
- The two axes divide the coordinate plane into four **quadrants**.



An **ordered pair** is a pair of numbers that gives the location of a point on a coordinate plane. The first number tells how far to the right (positive) or left (negative) the point is located from the origin. The second number tells how far up (positive) or down (negative) the point is located from the origin.

The numbers in an ordered pair are called **coordinates**. The first number is the **x-coordinate** and the second number is the **y-coordinate**.



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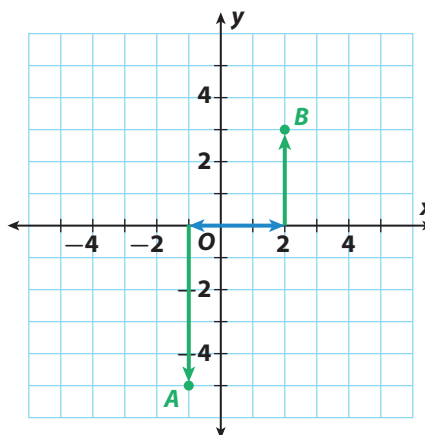
EXAMPLE 1

COMMON CORE 6.NS.6c, 6.NS.6b

Identify the coordinates of each point. Name the quadrant where each point is located.

Point *A* is 1 unit *left* of the origin, and 5 units *down*. It has x-coordinate -1 and y-coordinate -5 , written $(-1, -5)$. It is located in Quadrant III.

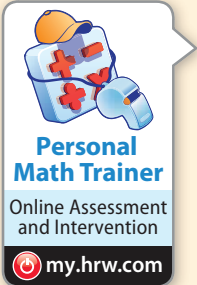
Point *B* is 2 units *right* of the origin, and 3 units *up*. It has x-coordinate 2 and y-coordinate 3 , written $(2, 3)$. It is located in Quadrant I.



Reflect

1. If both coordinates of a point are negative, in which quadrant is the point located? _____
2. Describe the coordinates of all points in Quadrant I.

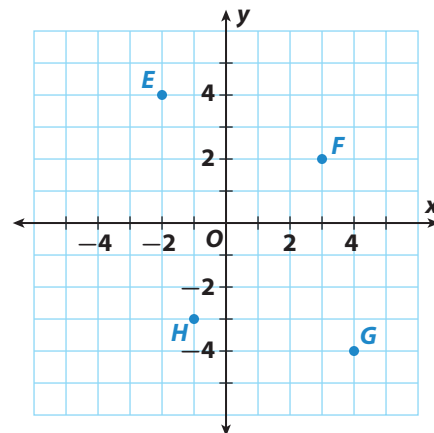
3. **Communicate Mathematical Ideas** Explain why $(-3, 5)$ represents a different location than $(3, 5)$.



YOUR TURN

Identify the coordinates of each point. Name the quadrant where each point is located.

4. G _____
E _____
5. F _____
H _____



Graphing Points in the Coordinate Plane

Points that are located on the axes are not located in any quadrant. Points on the x-axis have a y-coordinate of 0, and points on the y-axis have an x-coordinate of 0.



EXAMPLE 2

COMMON CORE 6.NS.6c, 6.NS.6

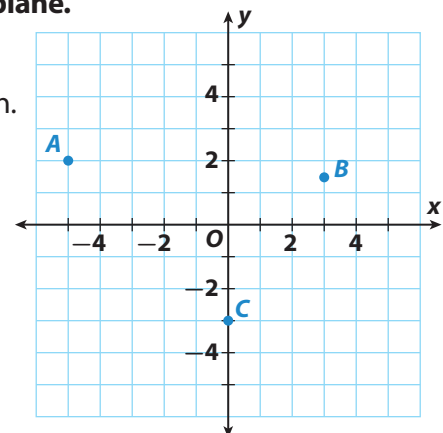
Graph and label each point on the coordinate plane.

$$A(-5, 2), B(3, 1.5), C(0, -3)$$

Point A is 5 units *left* and 2 units *up* from the origin.

Point B is 3 units *right* and 1.5 units *up* from the origin. Graph the point halfway between $(3, 1)$ and $(3, 2)$.

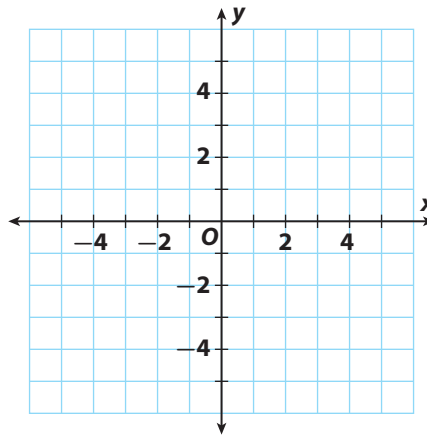
Point C is 3 units *down* from the origin. Graph the point on the y-axis.



YOUR TURN

Graph and label each point on the coordinate plane.

6. $P(-4, 2)$
7. $Q(3, 2.5)$
8. $R(-4.5, -5)$
9. $S(4, -5)$
10. $T(-2.5, 0)$



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Reading Scales on Axes

The *scale* of an axis is the number of units that each grid line represents. So far, the graphs in this lesson have a scale of 1 unit, but graphs frequently use other units.

EXAMPLE 3



COMMON
CORE 6.NS.8

The graph shows the location of a city. It also shows the location of Gary's and Jen's houses. The scale on each axis represents miles.

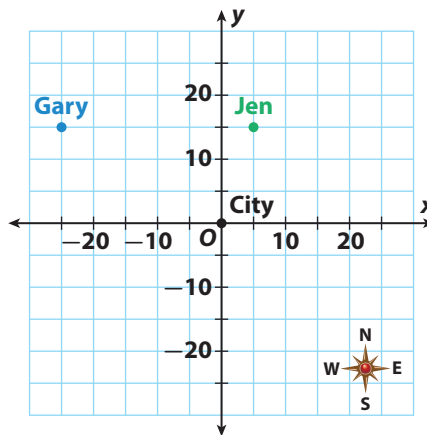
- A** Use the scale to describe Gary's location relative to the city.

Each grid square is 5 miles on a side.

Gary's house is at $(-25, 15)$, which is 25 miles west and 15 miles north of the city.

- B** Describe the location of Jen's house relative to Gary's house.

Jen's house is located 6 grid squares to the right of Gary's house. Since each grid square is 5 miles on a side, her house is $6 \cdot 5 = 30$ miles from Gary's.



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Math Talk

Mathematical Practices

How are north, south, east, and west represented on the graph in Example 3?

YOUR TURN

Use the graph in the Example.

11. Ted lives 20 miles south and 20 miles west of the city represented on the graph in Example 3. His brother Ned lives 50 miles north of Ted's house. Give the coordinates of each brother's house.



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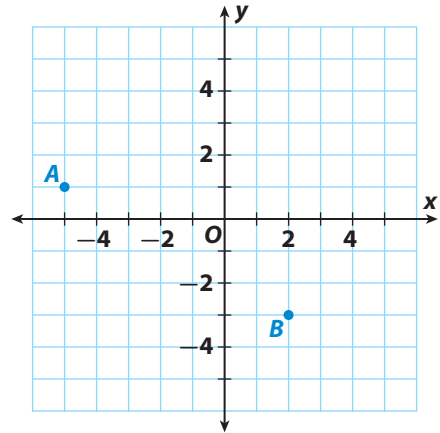
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Guided Practice

Identify the coordinates of each point in the coordinate plane.
Name the quadrant where each point is located. (Example 1)

- Point A is 5 units _____ of the origin and
1 unit _____ from the origin.
Its coordinates are _____. It is in quadrant _____.
- Point B is _____ units right of the origin
and _____ units down from the origin.
Its coordinates are _____. It is in quadrant _____.



Graph and label each point on the coordinate plane above. (Example 2)

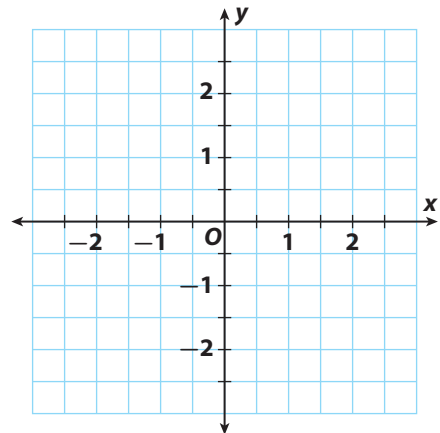
- Point C at $(-3.5, 3)$
- Point D at $(5, 0)$

For 5–7, use the coordinate plane shown. (Example 3)

- Describe the scale of the graph.

- Plot point A at $(-\frac{1}{2}, 2)$.
- Plot point B at $(2\frac{1}{2}, -2)$.

- Vocabulary** Describe how an ordered pair represents a point on a coordinate plane. Include the terms x-coordinate, y-coordinate, and origin in your answer.




ESSENTIAL QUESTION CHECK-IN

- Give the coordinates of one point in each of the four quadrants, one point on the x-axis, and one point on the y-axis.

12.1 Independent Practice

COMMON CORE 6.NS.6, 6.NS.6b, 6.NS.6c, 6.NS.8



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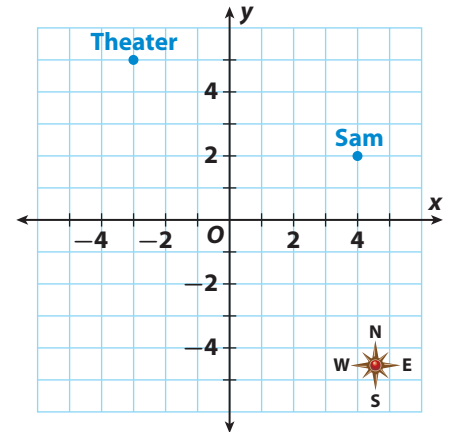
For 10–13, use the coordinate plane shown. Each unit represents 1 kilometer.

10. Write the ordered pairs that represent the location of Sam and the theater.

11. Describe Sam’s location relative to the theater.

12. Sam wants to meet his friend Beth at a restaurant before they go to the theater. The restaurant is 9 km south of the theater. Plot and label a point representing the restaurant. What are the coordinates of the point?

13. Beth describes her current location: “I’m directly south of the theater, halfway to the restaurant.” Plot and label a point representing Beth’s location. What are the coordinates of the point?

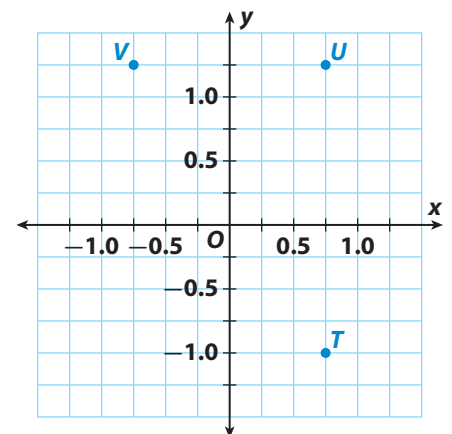


For 14–15, use the coordinate plane shown.

14. Find the coordinates of points T , U , and V .

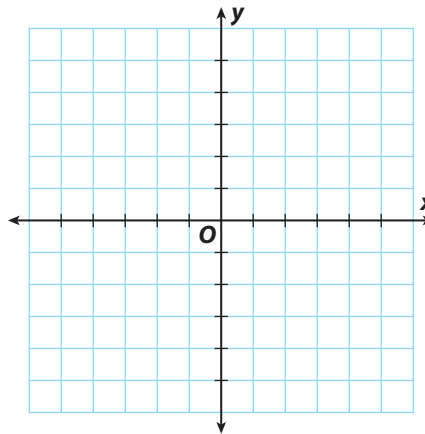
15. Points T , U , and V are the vertices of a rectangle. Point W is the fourth vertex. Plot point W and give its coordinates.

16. Explain the Error Janine tells her friend that ordered pairs that have an x -coordinate of 0 lie on the x -axis. She uses the origin as an example. Describe Janine’s error. Use a counterexample to explain why Janine’s statement is false.





- 17. Critical Thinking** Choose scales for the coordinate plane shown so that you can graph the points $J(2, 40)$, $K(3, 10)$, $L(3, -40)$, $M(-4, 50)$, and $N(-5, -50)$. Explain why you chose the scale for each axis.



- 18. Communicate Mathematical Ideas** Edgar wants to plot the ordered pair $(1.8, -1.2)$ on a coordinate plane. On each axis, one grid square equals 0.1. Starting at the origin, how can Edgar find $(1.8, -1.2)$?

- 19. Represent Real-World Problems** Zach graphs some ordered pairs in the coordinate plane. The x -values of the ordered pairs represent the number of hours since noon, and the y -values represent the temperature at that time.

- a.** In which quadrants could Zach graph points? Explain your thinking.

- b.** In what part of the world and at what time of year might Zach collect data so that the points he plots are in Quadrant IV?

LESSON 12.2 Independent and Dependent Variables in Tables and Graphs

COMMON CORE 6.EE.9

Use variables to represent two quantities in a real-world problem that change in relationship to one another; ... Analyze the relationship between the dependent and independent variables...



ESSENTIAL QUESTION

How can you identify independent and dependent quantities from tables and graphs?

EXPLORE ACTIVITY 1



COMMON CORE 6.EE.9

Identifying Independent and Dependent Quantities from a Table

Many real-world situations involve two variable quantities in which one quantity depends on the other. The quantity that depends on the other quantity is called the **dependent variable**, and the quantity it depends on is called the **independent variable**.

A freight train moves at a constant speed. The distance y in miles that the train has traveled after x hours is shown in the table.

Time x (h)	0	1	2	3
Distance y (mi)	0	50	100	150

- A** What are the two quantities in this situation?

Which of these quantities depends on the other?

What is the independent variable? _____

What is the dependent variable? _____

- B** How far does the train travel each hour? _____

The relationship between the distance traveled by the train and the time in hours can be represented by an equation in two variables.

Distance traveled (miles)	=	Distance traveled per hour	·	Time (hours)
↓		↓		↓
y	=	50	·	x

EXPLORE ACTIVITY (cont'd)

Reflect

1. **Analyze Relationships** Describe how the value of the independent variable is related to the value of the dependent variable. Is the relationship additive or multiplicative?

2. What are the units of the independent variable and of the dependent variable?

3. A rate is used in the equation. What is the rate?

EXPLORE ACTIVITY 2



COMMON
CORE

6.EE.9

Identifying Independent and Dependent Variables from a Graph

In Explore Activity 1, you used a table to represent a relationship between an independent variable (time) and a dependent variable (distance). You can also use a graph to show this relationship.

An art teacher has 20 pounds of clay but wants to buy more clay for her class. The amount of clay x purchased by the teacher and the amount of clay y available for the class are shown on the graph.

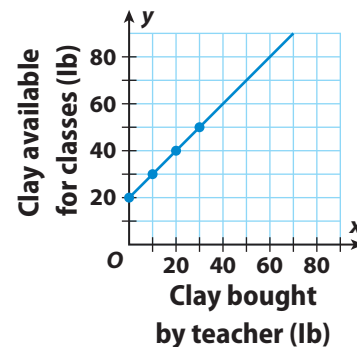
- A** If the teacher buys 10 more pounds of clay, how many pounds will be available for the art class? _____ lb

If the art class has a total of 50 pounds of clay available, how many pounds of clay did the teacher buy?

How can you use the graph to find this information?



Clay Used in Art Class



- B** What are the two quantities in this situation?

Which of these quantities depends on the other?

What is the independent variable? _____

What is the dependent variable? _____

- C** The relationship between the amount of clay purchased by the teacher and the amount of clay available to the class can be represented by an equation in two variables.

Amount of clay available (pounds)	=	Current amount of clay (pounds)	+	Amount of clay purchased (pounds)
↓		↓		↓
y	=	20	+	x

- D** Describe in words how the value of the independent variable is related to the value of the dependent variable.

Reflect

- 4.** In this situation, the same units are used for the independent and dependent variables. How is this different from the situation involving the train in the first Explore?

- 5. Analyze Relationships** Tell whether the relationship between the independent variable and the dependent variable is a multiplicative or an additive relationship.

- 6.** What are the units of the independent variable, and what are the units of the dependent variable?

independent variable: _____; dependent variable: _____



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Describing Relationships Between Independent and Dependent Variables

Thinking about how one quantity depends on another helps you identify which quantity is the independent variable and which quantity is the dependent variable. In a graph, the independent variable is usually shown on the horizontal axis and the dependent variable on the vertical axis.

EXAMPLE 1



COMMON CORE

6.EE.9

- A** The table shows a relationship between two variables, x and y . Describe a possible situation the table could represent. Describe the independent and dependent variables in the situation.

Independent variable, x	0	1	2	3
Dependent variable, y	10	11	12	13

As x increases by 1, y increases by 1. The relationship is additive.

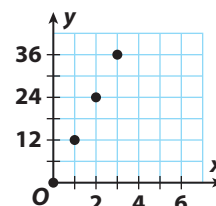
The value of y is always 10 units greater than the value of x .

The table could represent Jina's savings if she starts with \$10 and adds \$1 to her savings every day.

The independent variable, x , is the number of days she has been adding money to her savings.

The dependent variable, y , is her savings after x days.

- B** The graph shows a relationship between two variables. Describe a possible situation that the graph could represent. Describe the independent and dependent variables.



As x increases by 1, y increases by 12. The relationship is multiplicative. The value of y is always 12 times the value of x .

The graph could represent the number of eggs in cartons that each hold 12 eggs.

The independent variable, x , is the number of cartons.

The dependent variable, y , is the total number of eggs.

Reflect

7. What are other possible situations that the table and graph in the Examples could represent?

YOUR TURN

Describe real-world values that the variables could represent. Describe the relationship between the independent and dependent variables.

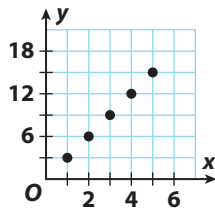
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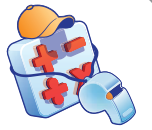
x	0	1	2	3
y	15	16	17	18

9.

x	0	1	2	3	4
y	0	16	32	48	64

10.





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Guided Practice

1. A boat rental shop rents paddleboats for a fee plus an additional cost per hour. The cost of renting for different numbers of hours is shown in the table.

Time (hours)	0	1	2	3
Cost (\$)	10	11	12	13

What is the independent variable, and what is the dependent variable?
How do you know? ([Explore Activity 1](#))

2. A car travels at a constant rate of 60 miles per hour. ([Explore Activity 1](#))

Time x (h)	0	1	2	3
Distance y (mi)				

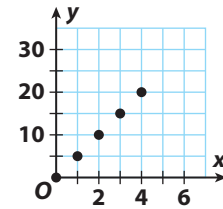
- a. Complete the table.
- b. What is the independent variable, and what is the dependent?

- c. Describe how the value of the independent variable is related to the value of the dependent variable.

Use the graph to answer the questions.

3. Describe in words how the value of the independent variable is related to the value of the dependent variable. ([Explore Activity 2](#))

4. Describe a real-world situation that the graph could represent. ([Example 1](#))




ESSENTIAL QUESTION CHECK-IN

5. How can you identify the dependent and independent variables in a real-world situation modeled by a graph?

12.2 Independent Practice

COMMON CORE 6.EE.9

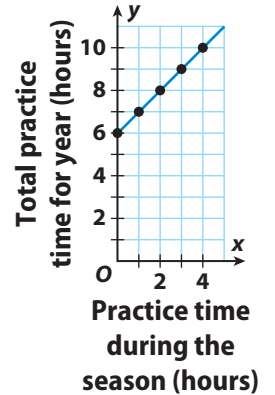


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6. The graph shows the relationship between the hours a soccer team practiced after the season started and their total practice time for the year.



a. How many hours did the soccer team practice before the season began?

b. What are the two quantities in this situation?

c. What are the dependent and independent variables?

d. Is the relationship between the variables additive or multiplicative? Explain.

e. **Analyze Relationships** Describe the relationship between the quantities in words.

7. **Multistep** Teresa is buying glitter markers to put in gift bags. The table shows the relationship between the number of gift bags and the number of glitter markers she needs to buy.

Number of gift bags, x	0	1	2	3
Number of markers, y	0	5	10	15

a. What is the dependent variable? _____

b. What is the independent variable? _____

c. Is the relationship additive or multiplicative? Explain.

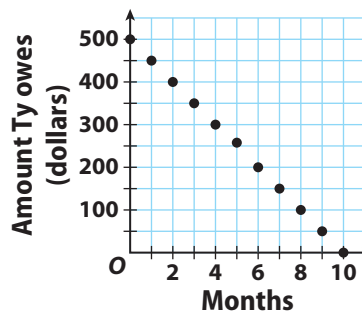
d. Describe the relationship between the quantities in words.

8. Ty borrowed \$500 from his parents. The graph shows how much he owes them each month if he pays back a certain amount each month.

a. Describe the relationship between the number of months and the amount Ty owes. Identify an independent and dependent variable and explain your thinking.

b. How long will it take Ty to pay back his parents?

Ty's Loan Payments



FOCUS ON HIGHER ORDER THINKING

9. **Error Analysis** A discount store has a special: 8 cans of juice for a dollar. A shopper decides that since the number of cans purchased is 8 times the number of dollars spent, the cost is the independent variable and the number of cans is the dependent variable. Do you agree? Explain.

10. **Analyze Relationships** Provide an example of a real-world relationship where there is no clear independent or dependent variable. Explain.

Work Area

LESSON 12.3 Writing Equations from Tables

COMMON CORE 6.EE.9

... write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. ...



ESSENTIAL QUESTION

How can you use an equation to show a relationship between two variables?

EXPLORE ACTIVITY



COMMON CORE 6.EE.9

Writing an Equation to Represent a Real-World Relationship

Many real-world situations involve two variable quantities in which one quantity depends on the other. This type of relationship can be represented by a table. You can also use an equation to model the relationship.



The table shows how much Amanda earns for walking 1, 2, or 3 dogs. Use the table to determine how much Amanda earns per dog. Then write an equation that models the relationship between number of dogs walked and earnings. Use your equation to complete the table.

Dogs walked	1	2	3	5	10	20
Earnings	\$8	\$16	\$24			

For 1 dog, Amanda earns $1 \cdot 8 = \$8$.
For 2 dogs, she earns $2 \cdot 8 = \$16$.

- A** For each column, compare the number of dogs walked and earnings. What is the pattern?

- B** Based on the pattern, Amanda earns \$ _____ for each dog she walks.
- C** Write an equation that relates the number of dogs Amanda walks to the amount she earns. Let e represent earnings and d represent dogs.

- D** Use your equation to complete the table for 5, 10, and 20 walked dogs.
- E** Amanda's earnings depend on _____.

Reflect

1. **What If?** If Amanda changed the amount earned per dog to \$11, what equation could you write to model the relationship between number of dogs walked and earnings? _____



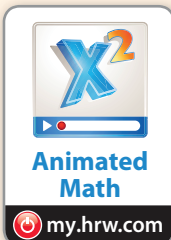
Math On the Spot

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Writing an Equation Based on a Table

The relationship between two variables where one variable depends on the other can be represented in a table or by an equation. An equation expresses the dependent variable in terms of the independent variable.

When there is no real-world situation to consider, we usually say x is the independent variable and y is the dependent variable. The value of y depends on the value of x .



Animated Math

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EXAMPLE 1

COMMON CORE 6.EE.9

Write an equation that expresses y in terms of x .

A

x	1	2	3	4	5
y	0.5	1	1.5	2	2.5

STEP 1 Compare the x - and y -values to find a pattern.

Each y -value is $\frac{1}{2}$, or 0.5 times, the corresponding x -value.

STEP 2 Use the pattern to write an equation expressing y in terms of x .

$y = 0.5x$

B

x	2	4	6	8	10
y	5	7	9	11	13

STEP 1 Compare the x - and y -values to find a pattern.

Each y -value is 3 more than the corresponding x -value.

STEP 2 Use the pattern to write an equation expressing y in terms of x .

$y = x + 3$

Math Talk

Mathematical Practices

How can you check that your equations are correct?

YOUR TURN

For each table, write an equation that expresses y in terms of x .

2.

x	12	11	10
y	10	9	8

3.

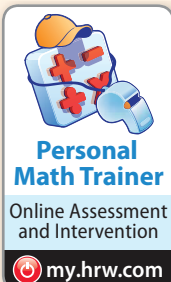
x	10	12	14
y	25	30	35

4.

x	5	4	3
y	10	9	8

5.

x	0	1	2
y	0	2	4



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Using Tables and Equations to Solve Problems

You can use tables and equations to solve real-world problems.



Math On the Spot

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EXAMPLE 2

Problem Solving

COMMON CORE 6.EE.9

A certain percent of the sale price of paintings at a gallery will be donated to charity. The donation will be \$50 if a painting sells for \$200. The donation will be \$75 if a painting sells for \$300. Find the amount of the donation if a painting sells for \$1,200.



Analyze Information

You know the donation amount when the sale price of a painting is \$200 and \$300. You need to find the donation amount if a painting sells for \$1,200.

Formulate a Plan

You can make a table to help you determine the relationship between sale price and donation amount. Then you can write an equation that models the relationship. Use the equation to find the unknown donation amount.

Solve

Make a table.

Sale price (\$)	200	300
Donation amount (\$)	50	75

$$\frac{50}{200} = \frac{50 \div 2}{200 \div 2} = \frac{25}{100} = 25\%$$

$$\frac{75}{300} = \frac{75 \div 3}{300 \div 3} = \frac{25}{100} = 25\%$$

One way to determine the relationship between sale price and donation amount is to find the percent.

Write an equation. Let p represent the sale price of the painting. Let d represent the donation amount to charity.

The donation amount is equal to 25% of the sale price.

$$d = 0.25 \cdot p$$

p is the independent variable; its value does not depend on any other value. d is the dependent variable; its value depends on the price of the painting.

Find the donation amount when the sale price is \$1,200.

$$d = 0.25 \cdot p$$

$$d = 0.25 \cdot 1,200 \quad \text{Substitute \$1,200 for the sale price of the painting.}$$

$$d = 300 \quad \text{Simplify to find the donation amount.}$$

When the sale price is \$1,200, the donation to charity is \$300.

Justify and Evaluate

Substitute values from the table for p and d to check that they are solutions of the equation $d = 0.25 \cdot p$. Then check your answer of \$300 by substituting for d and solving for p .

$$\begin{aligned} d &= 0.25 \cdot p \\ d &= 0.25 \cdot 200 \\ d &= 50 \end{aligned} \quad \checkmark$$

$$\begin{aligned} d &= 0.25 \cdot p \\ d &= 0.25 \cdot 300 \\ d &= 75 \end{aligned} \quad \checkmark$$

$$\begin{aligned} d &= 0.25 \cdot p \\ 300 &= 0.25 \cdot p \\ p &= 1,200 \end{aligned} \quad \checkmark$$

YOUR TURN

6. When Ryan is 10, his brother Kyle is 15. When Ryan is 16, Kyle will be 21. When Ryan is 21, Kyle will be 26. Write and solve an equation to find Kyle's age when Ryan is 52.

Guided Practice

Write an equation to express y in terms of x . (Explore Activity, Example 1)

1.

x	10	20	30	40
y	6	16	26	36

2.

x	0	1	2	3
y	0	4	8	12

3.

x	4	6	8	10
y	7	9	11	13

4.

x	12	24	36	48
y	2	4	6	8

5. Jameson downloaded one digital song for \$1.35, two digital songs for \$2.70, and 5 digital songs for \$6.75. Write and solve an equation to find the cost to download 25 digital songs. (Example 2)

Songs downloaded	1	2	5	10
Total cost (\$)	1.35			

Number of songs = n ; Cost = _____

The total cost of 25 songs is _____

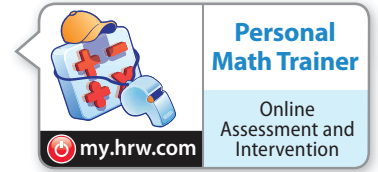


ESSENTIAL QUESTION CHECK-IN

6. Explain how to use a table to write an equation that represents the relationship in the table.

12.3 Independent Practice

COMMON CORE 6.EE.9



7. Vocabulary What does it mean for an equation to express y in terms of x ?

8. The length of a rectangle is 2 inches more than twice its width. Write an equation relating the length l of the rectangle to its width w .

9. Look for a Pattern Compare the y -values in the table to the corresponding x -values. What pattern do you see? How is this pattern used to write an equation that represents the relationship between the x - and y -values?

x	20	24	28	32
y	5	6	7	8

10. Explain the Error A student modeled the relationship in the table with the equation $x = 4y$. Explain the student's error. Write an equation that correctly models the relationship.

x	2	4	6	8
y	8	16	24	32

11. Multistep Marvin earns \$8.25 per hour at his summer job. He wants to buy a video game system that costs \$206.25.

a. Write an equation to model the relationship between number of hours worked h and amount earned e .

b. Solve your equation to find the number of hours Marvin needs to work in order to afford the video game system.

- 12. Communicate Mathematical Ideas** For every hour that Noah studies, his test score goes up 3 points. Explain which is the independent variable and which is the dependent variable. Write an equation modeling the relationship between hours studied h and the increase in Noah's test score s .



FOCUS ON HIGHER ORDER THINKING

- 13. Make a Conjecture** Compare the y -values in the table to the corresponding x -values. Determine whether there is an additive relationship or a multiplicative relationship between x and y . If possible, write an equation modeling the relationship. If not explain why.

x	1	3	5	7
y	3	6	8	21

- 14. Represent Real-World Problems** Describe a real-world situation in which there is an additive or multiplicative relationship between two quantities. Make a table that includes at least three pairs of values. Then write an equation that models the relationship between the quantities.

- 15. Critical Thinking** Georgia knows that there is either an additive or multiplicative relationship between x and y . She only knows a single pair of data values. Explain whether Georgia has enough information to write an equation that models the relationship between x and y .

Work Area

LESSON 12.4

Representing Algebraic Relationships in Tables and Graphs

COMMON CORE 6.EE.9

...Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.



ESSENTIAL QUESTION

How can you use verbal descriptions, tables, and graphs to represent algebraic relationships?

EXPLORE ACTIVITY 1



COMMON CORE 6.EE.9

Representing Algebraic Relationships

Angie's walking speed is 5 kilometers per hour, and May's is 4 kilometers per hour. Show how the distance each girl walks is related to time.

- A** For each girl, make a table comparing time and distance.

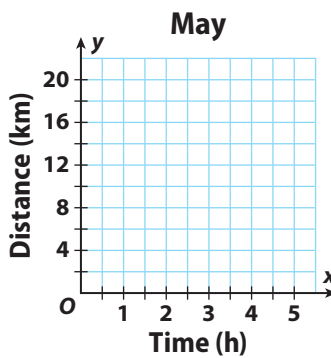
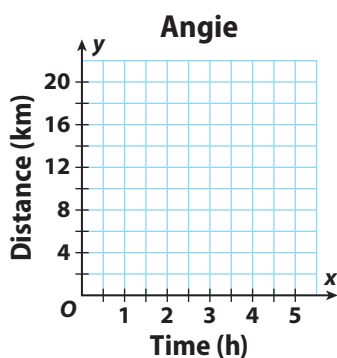
Time (h)	0	1	2	3	4
Angie's distance (km)	0	5	10		

For every hour Angie walks, she travels 5 km.

Time (h)	0	1	2	3	4
May's distance (km)	0	4	8		

For every hour May walks, she travels 4 km.

- B** For each girl, make a graph showing her distance y as it depends on time x . Plot points from the table and connect them with a line. Write an equation for each girl that relates distance y to time x .



Angie's equation: _____

May's equation: _____

Math Talk

Mathematical Practices

Why does it make sense to connect the points in each graph?

Reflect

1. **Analyze Relationships** How can you use the tables to determine which girl is walking faster? How can you use the graphs?

Writing an Equation from a Graph

Cherise pays the entrance fee to visit a museum, then buys souvenirs at the gift shop. The graph shows the relationship between the total amount she spends at the museum and the amount she spends at the gift shop. Write an equation to represent the relationship.

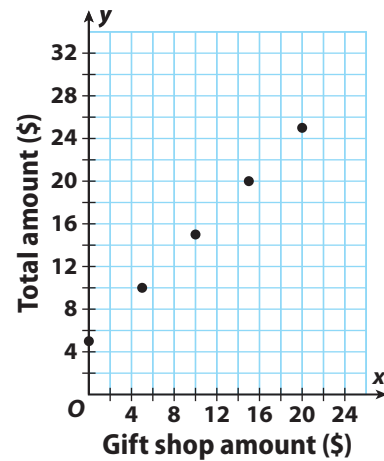


- A** Read the ordered pairs from the graph. Use them to complete a table comparing total spent y to amount spent at the gift shop x .

Gift shop amount (\$)	0	5	10	15	
Total amount (\$)	5	10			

- B** What is the pattern in the table?

- C** Write an equation that expresses the total amount spent, y , in terms of the amount spent at the gift shop, x .



Reflect

- 2. Communicate Mathematical Ideas** Identify the dependent and independent quantities in this situation.

- 3. Multiple Representations** Draw a line through the points on the graph. Find the point that represents Cherise spending \$18 at the gift shop. Use this point to find the total she would spend if she spent \$18 at the gift shop. Then use your equation from **C** to verify your answer.

Graphing an Equation

An ordered pair (x, y) that makes an equation like $y = x + 1$ true is called a **solution** of the equation. The graph of an equation represents all the ordered pairs that are solutions.



EXAMPLE 1

COMMON CORE 6.EE.9

Graph each equation.

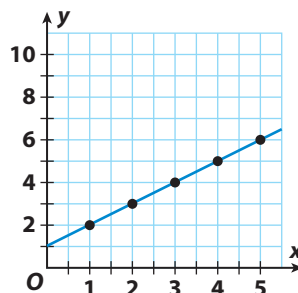
A $y = x + 1$

STEP 1 Make a table of values. Choose some values for x and use the equation to find the corresponding values for y .

STEP 2 Plot the ordered pairs from the table.

STEP 3 Draw a line through the plotted points to represent all of the ordered pair solutions of the equation.

x	$x + 1 = y$	(x, y)
1	$1 + 1 = 2$	(1, 2)
2	$2 + 1 = 3$	(2, 3)
3	$3 + 1 = 4$	(3, 4)
4	$4 + 1 = 5$	(4, 5)
5	$5 + 1 = 6$	(5, 6)



Math Talk

Mathematical Practices

Is the ordered pair $(3.5, 4.5)$ a solution of the equation $y = x + 1$? Explain.

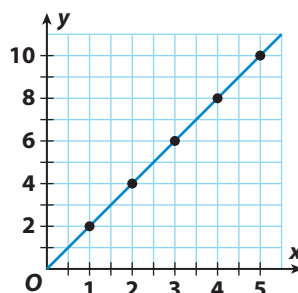
B $y = 2x$

STEP 1 Make a table of values. Choose some values for x and use the equation to find the corresponding values for y .

STEP 2 Plot the ordered pairs from the table.

STEP 3 Draw a line through the plotted points to represent all of the ordered pair solutions of the equation.

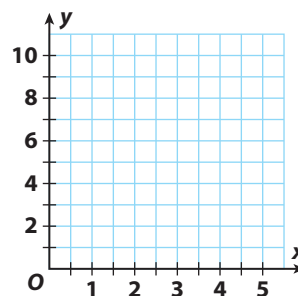
x	$2x = y$	(x, y)
1	$2 \times 1 = 2$	(1, 2)
2	$2 \times 2 = 4$	(2, 4)
3	$2 \times 3 = 6$	(3, 6)
4	$2 \times 4 = 8$	(4, 8)
5	$2 \times 5 = 10$	(5, 10)



YOUR TURN

4. Graph $y = x + 2.5$.

x	$x + 2.5 = y$	(x, y)

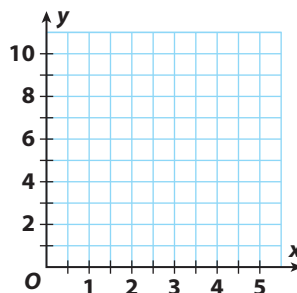


Guided Practice

Frank mows lawns in the summer to earn extra money. He can mow 3 lawns every hour he works. (Explore Activity 1 and Explore Activity 2)

- Make a table to show the relationship between the number of hours Frank works, x , and the number of lawns he mows, y . Graph the relationship and write an equation. Label the axes of your graph.

Hours worked	Lawns mowed
0	
1	

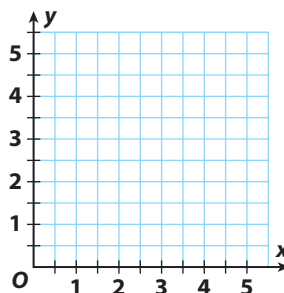


Graph $y = 1.5x$. (Example 1)

- Make a table to show the relationship.

x				
y				

- Plot the points and draw a line through them.




ESSENTIAL QUESTION CHECK-IN

- How can a table represent an algebraic relationship between two variables?

12.4 Independent Practice

COMMON CORE 6.EE.9



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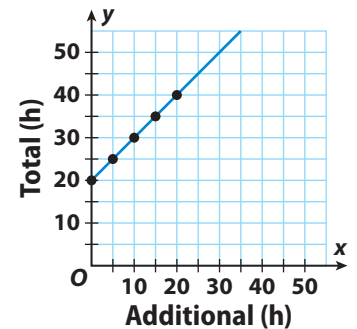
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Students at Mills Middle School are required to work a certain number of community service hours. The table shows the numbers of additional hours several students worked beyond their required hours, as well as the total numbers of hours worked.

5. Read the ordered pairs from the graph to make a table.

Additional hours					
Total hours					



6. Write an equation that expresses the total hours in terms of the additional hours.

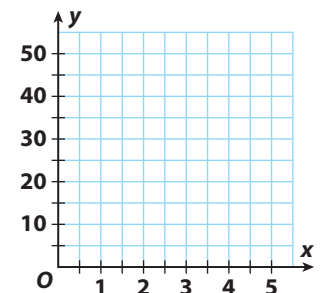
7. **Analyze Relationships** How many community service hours are students required to work? Explain.

Beth is using a map. Let x represent a distance in centimeters on the map. To find an actual distance y in kilometers, Beth uses the equation $y = 8x$.

8. Make a table comparing a distance on the map to the actual distance.

Map distance (cm)					
Actual distance (km)					

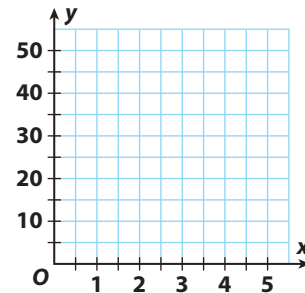
9. Make a graph that compares the map distance to the actual distance. Label the axes of the graph.
10. **Critical Thinking** The actual distance between Town A and Town B is 64 kilometers. What is the distance on Beth's map? Did you use the graph or the equation to find the answer? Why?



11. Multistep The equation $y = 9x$ represents the total cost y for x movie tickets. Label the axes of the graph.

a. Make a table and a graph to represent the relationship between x and y .

Number of tickets, x					
Total cost (\$), y					



b. Critical Thinking In this situation, which quantity is dependent and which is independent? Justify your answer.

c. Multiple Representations Eight friends want to go see a movie. Would you prefer to use an equation, a table, or a graph to find the cost of 8 movie tickets? Explain how you would use your chosen method to find the cost.

H.O.T. FOCUS ON HIGHER ORDER THINKING

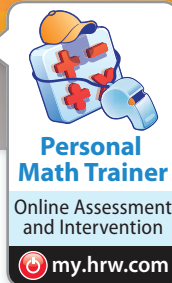
12. Critical Thinking Suppose you graph $y = 5x$ and $y = x + 500$ on the same coordinate plane. Which line will be steeper? Why?

13. Persevere in Problem Solving Marcus plotted the points $(0, 0)$, $(6, 2)$, $(18, 6)$, and $(21, 7)$ on a graph. He wrote an equation for the relationship. Find another ordered pair that could be a solution of Marcus's equation. Justify your answer.

14. Error Analysis The cost of a personal pizza is \$4. A drink costs \$1. Anna wrote the equation $y = 4x + 1$ to represent the relationship between total cost y of buying x meals that include one personal pizza and one drink. Describe Anna's error and write the correct equation.

Work Area

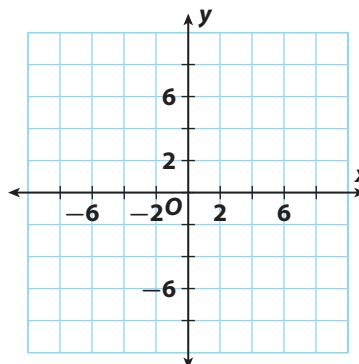
Ready to Go On?



12.1 Graphing on the Coordinate Plane

Graph each point on the coordinate plane.

- 1. $A(-2, 4)$
- 2. $B(3, 5)$
- 3. $C(6, -4)$
- 4. $D(-3, -5)$
- 5. $E(7, 2)$
- 6. $F(-4, 6)$



12.2 Independent and Dependent Variables in Tables and Graphs

- 7. Jon buys packages of pens for \$5 each. Identify the independent and dependent variables in the situation.

12.3 Writing Equations from Tables

Write an equation that represents the data in the table.

8.

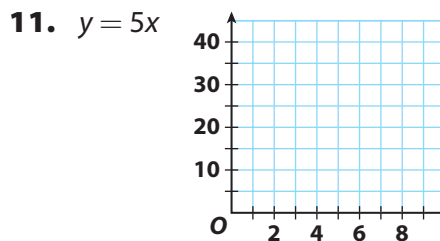
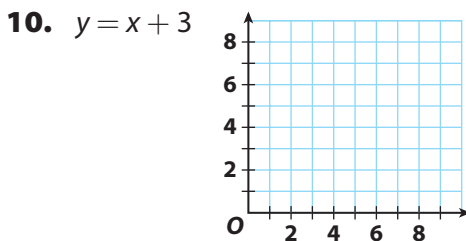
x	3	5	8	10
y	21	35	56	70

9.

x	5	10	15	20
y	17	22	27	32

12.4 Representing Algebraic Relationships in Tables and Graphs

Graph each equation.



ESSENTIAL QUESTION

- 12. How can you write an equation in two variables to solve a problem?

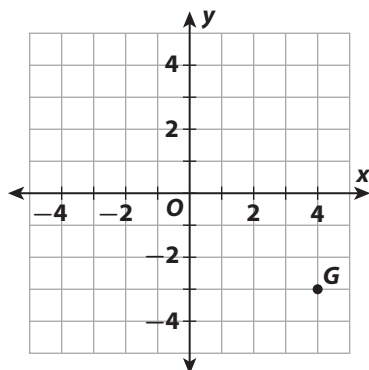


Assessment Readiness



Selected Response

1. What are the coordinates of point G on the coordinate grid below?



- (A) (4, 3) (C) (-4, 3)
 (B) (4, -3) (D) (-4, -3)
2. A point is located in quadrant II of a coordinate plane. Which of the following could be the coordinates of that point?

- (A) (-5, -7) (C) (-5, 7)
 (B) (5, 7) (D) (5, -7)

3. Matt had 5 library books. He checked 1 additional book out every week without returning any books. Which equation describes the number of books he has, y , after x weeks?

- (A) $y = 5x$ (C) $y = 1 + 5x$
 (B) $y = 5 - x$ (D) $y = 5 + x$

4. Stewart is playing a video game. He earns the same number of points for each prize he captures. He earned 1,200 points for 6 prizes, 2,000 points for 10 prizes, and 2,600 points for 13 prizes. Which is the dependent variable in the situation?

- (A) the number of prizes captured
 (B) the number of points earned
 (C) the number of hours
 (D) the number of prizes available

5. Which point is *not* on the graph of the equation $y = 10 + x$?

- (A) (0, 10) (C) (8, 2)
 (B) (3, 13) (D) (5, 15)

6. Amy gets paid by the hour. Her sister helps. As shown, Amy gives her sister part of her earnings. Which equation represents Amy's pay when her sister's pay is \$13?

Amy's pay in dollars	10	20	30	40
Sister's pay in dollars	2	4	6	8

- (A) $y = \frac{13}{5}$ (C) $5 = 13y$
 (B) $13 = \frac{x}{5}$ (D) $13 = 5x$

Mini-Task

7. The table compares the ages, in years, of two cousins.

Ann's age, x	4	8	12
Tom's age, y	8	12	16

- a. Write an equation that compares Tom's and Ann's ages.

- b. Draw a graph to represent the equation.

