Area and Polygons



ESSENTIAL QUESTION

How can you find the area of an irregular polygon using area formulas?



LESSON 13.1 Area of Quadrilaterals

LESSON 13.2 Area of Triangles



6.G.1, 6.EE.7

LESSON 13.4 Area of Polygons

C HOU





my.hrw.com

Go digital with your write-in student edition, accessible on any device.



my.hrw.com

Math On the Spot

Scan with your smart phone to jump directly to the online edition, video tutor, and more.



Real-World Video Quilting, painting, and other art forms use familiar geometric shapes, such as triangles and rectangles. To buy enough supplies for a project, you need to find or estimate the areas of each shape in the project.

Animated Math

Interactively explore key concepts to see how math works.



Personal Math Trainer

Get immediate feedback and help as you work through practice sets.

Are Ready

Complete these exercises to review skills you will need for this module.



Inverse Operations

EXAMPLES

7 <i>k</i> = 35	k is multiplied by 7.	<i>k</i> + 7 = 9	7 is added to k.
$\frac{7k}{7} = \frac{35}{7}$	Use the inverse	k + 7 - 7 = 9 - 7	Use the inverse
<i>k</i> = 5	operation, aivision.	<i>k</i> = 2	operation, subt

the inverse ration, subtraction.

Solve each equation using the inverse operation.

1.	9 <i>p</i> = 54	2.	m - 15 = 9	3.	$\frac{b}{8} = 4$	4.	z + 17 = 23	
----	-----------------	----	------------	----	-------------------	----	-------------	--

Metric Units

EX/

MPLE	6 m = cm	Multiply to go from a larger unit to a smaller unit.
	6 m = 600 cm	
	4,000 mL = L	Divide to go from a smaller unit to a larger unit.
	4,000 mL = 4 L	v

Convert to the given units.

5. $64 \text{ m} = _$ cm **6.** $500 \text{ g} = _$ kg **7.** $4.6 \text{ kL} = _$ L

Area of Squares and Rectangles



8. Find the area of a rectangle with a base of 5 feet and a height

of 9 ¹/₂ feet _____

Reading Start-Up

Visualize Vocabulary

Use the 🖌 words to complete the graphic. You will put one word in each oval.



Vocabulary

Review Words

- ✓ hexagon (*hexágono*)
- ✓ polygon (polígono regular)

quadrilateral (cuadrilátero) rectangular prism (prisma rectangular) regular polygon (polígono) right triangle (triángulo

- rectángulo)
- ✓ triangle (triángulo)

Preview Words parallelogram (paralelogramo) rhombus (rombo) trapezoid (trapecio)

Understand Vocabulary

Match the term on the left to the correct expression on the right.

1.	parallelogram	Α.	A quadrilateral in which all sides are congruent and opposite sides are parallel.
2.	trapezoid	В.	A quadrilateral in which opposite sides are parallel and congruent.
3.	rhombus	с.	A quadrilateral in which two sides are parallel.

Active Reading

Pyramid Before beginning the module, create a pyramid to help you organize what you learn. Label each side with one of the lesson titles from this module. As you study each lesson, write important ideas like vocabulary, properties, and formulas on the appropriate side.



MODULE 13 Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

COMMON 6.G.1

Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

What It Means to You

You will use the formula for the area of a figure to write an equation that can be used to solve a problem.

UNPACKING EXAMPLE 6.G.1

The Hudson Middle School wrestling team won the state tournament and was awarded a triangular pennant to display in the school gymnasium. The pennant has an area of 2.25 square meters. The base of the pennant is 1.5 meters long. Write an equation to find the height of the pennant.



An equation to find the height of the pennant is 2.25 = 0.75h.

COMMON 6.G.1

Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

What It Means to You

You will use formulas to find the area of irregular polygons.

UNPACKING EXAMPLE 6.G.1

John is measuring his room for new carpet. Find the area of the room.

Find the area of the rectangle.

 $A = bh = 15 \times 6 = 90 \, \text{ft}^2$

Find the area of the square.

 $A = s^2 = 6^2 = 36 \text{ ft}^2$

The total area is 90 $ft^2 + 36 ft^2 = 126 ft^2$.





Visit **my.hrw.com** to see all the **Common Core Standards** unpacked.

🕑 my.hrw.com



Lesson 13.1 371

EXPLORE ACTIVITY (cont'd)

Reflect

1. Find the area of the parallelogram.





Finding the Area of a Trapezoid

To find the formula for the area of a trapezoid, notice that two copies of the same trapezoid fit together to form a parallelogram. Therefore, the area of the trapezoid is $\frac{1}{2}$ the area of the parallelogram.



The height of the parallelogram is the same as the height of the trapezoid. The base of the parallelogram is the sum of the two bases of the trapezoid.

$$A = b \cdot h$$

$$\downarrow$$

$$A = (b_1 + b_2) \cdot h$$

6.G.1

Area of a Trapezoid

The area of a trapezoid is half its height multiplied by the sum of the lengths of its two bases.

$$A = \frac{1}{2}h(b_1 + b_2)$$

Rea **EXAMPLE 1** vorle

A section of a deck is in the shape of a trapezoid. What is the area of this section of the deck?

Does it matter which of

the trapezoid's bases is substituted for b_1 and which, is substituted for b_2 ? Why or why not?

 $b_2 = 39$ h = 16 $b_1 = 17$

Use the formula for area of a trapezoid.

$$A = \frac{1}{2}h(b_1 + b_2)$$

 $=\frac{1}{2}\cdot 16(17+39)$ Substitute.

 $=\frac{1}{2} \cdot 16(56)$ Add inside the parentheses.

Multiply $\frac{1}{2}$ and 16. $= 8 \cdot 56$

= 448 square feet Multiply.



 b_1

b,

Animated Math

my.hrw.com



2. Another section of the deck is also shaped like a trapezoid. For this section, the length of one base is 27 feet, and the length of the other base is 34 feet. The height is 12 feet. What is the area of this section of

the deck? A =____ft²

Finding the Area of a Rhombus

A **rhombus** is a quadrilateral in which all sides are congruent and opposite sides are parallel. A rhombus can be divided into four triangles that can then be rearranged into a rectangle.



d.

The base of the rectangle is the same length as one of the diagonals of the rhombus. The height of the rectangle is $\frac{1}{2}$ the length of the other diagonal.

$$A = b \cdot h$$

$$\downarrow \qquad \downarrow$$

$$A = d_1 \cdot \frac{1}{2}d_2$$

Area of a Rhombus

The area of a rhombus is half of the product of its two diagonals.

 $A = \frac{1}{2} d_1 d_2$

Real **EXAMPLE 2**

Cedric is constructing a kite in the shape of a rhombus. The spars of the kite measure 15 inches and 24 inches. How much fabric will Cedric need for the kite?

To determine the amount of fabric needed, find the area of the kite.

$$d_1 = 15$$
 $d_2 = 24$

Use the formula for area of a rhombus.

$$A = \frac{1}{2}d_1d_2$$
$$= \frac{1}{2}(15)(2$$

Substitute.

= 180 square inches

Multiply.











Find the area of each rhombus.

3.
$$d_1 = 35 \text{ m}; d_2 = 12 \text{ m}$$

 $A = _ m^2$
4. $d_1 = 9.5 \text{ in.}; d_2 = 14 \text{ in.}$
 $A = _ m^2$
5. $d_1 = 10 \text{ m}; d_2 = 18 \text{ m}$
 $A = _ m^2$
6. $d_1 = 8\frac{1}{4} \text{ ft}; d_2 = 40 \text{ ft}$
 $A = _ ft^2$

Guided Practice

1. Find the area of the parallelogram. (Explore Activity)

$$A = bh$$

$$= (\underline{\qquad})(\underline{\qquad})$$
$$= \underline{\qquad} in^2$$

2. Find the area of the trapezoid. (Example 1)



= _____ cm²

3. Find the area of the rhombus. (Example 2)

$$A = \frac{1}{2}d_1d_2$$
$$= \frac{1}{2}\left(\bigcirc\right)\left(\bigcirc\right)$$

= _____ in²



9 in.

13 in.

9 cm

15 cm

14 cm

ESSENTIAL QUESTION CHECK-IN

4. How can you find the areas of parallelograms, rhombuses, and trapezoids?

Class



32 m. What is the area of the rhombus?

15. Multistep A parking space shaped like a parallelogram has a base of 17 feet and a height is 9 feet. A car parked in the space is 16 feet long and 6 feet wide. How much of the parking space is not covered by the car?

H.O.T.

FOCUS ON HIGHER ORDER THINKING

16. Critique Reasoning Simon says that to find the area of a trapezoid, you can multiply the height by the top base and the height by the bottom base. Then add the two products together and divide the sum by 2. Is Simon correct? Explain your answer.

17. Multistep The height of a trapezoid is 8 in. and its area is 96 in.² One base of the trapezoid is 6 inches longer than the other base. What are the lengths of the bases? Explain how you found your answer.

18. Multiple Representations The diagonals of a rhombus are 12 in. and 16 in. long. The length of a side of the rhombus is 10 in. What is the height of the rhombus? Explain how you found your answer.

Work Area



EXPLORE ACTIVITY 2 6.G.1

Area of a Triangle

A Draw a large triangle on grid paper. Do not draw a right triangle.

B Cut out your triangle. Then trace around it to make a copy of your triangle. Cut out the copy.

Cut one of your triangles into two pieces by cutting through one angle directly across to the opposite side. Now you have three triangles — one large triangle and two smaller right triangles.



EXPLORE ACTIVITY 2 (cont'd)

When added together, the areas of the two smaller triangles

equal the ______ of the large triangle.

D Arrange the three triangles into a rectangle.

What fraction of the rectangle does the large

triangle represent? _____

The area of the rectangle is A = bh. What is the area

of the large triangle? *A* = _____

How does this formula compare to the formula for the area of a right triangle that you found in Explore Activity 1?



Reflect

2. Communicate Mathematical Ideas What type of angle is formed by the base and height of a triangle?





The area of one small equilateral triangle is 62.4 m².

Reflect

4. Persevere in Problem Solving What is the total area of one face of the pyramid? What is the total surface area of the faces of the pyramid, not counting the bottom? (Hint: the bottom of the pyramid is a square.)



YOUR TURN

5. Amy needs to order a shade for a triangular-shaped window that has a base of 6 feet and a height of 4 feet. What is the area of the shade?

Guided Practice





Class

Personal Math Trainer

13.2 Independent Practice 6.G.1

Find the area of each triangle.





- **8.** What is the area of a triangle that has a base of $15\frac{1}{4}$ in. and a height of 18 in.?
- **10.** A triangular plot of land has the dimensions shown in the diagram. What is the area of the land?



- **9.** A right triangle has legs that are 11 in. and 13 in. long. What is the area of the triangle?
- **11.** The front part of a tent has the dimensions shown in the diagram. What is the area of this part of the tent?



13. Critique Reasoning Monica has a triangular piece of fabric. The height of the triangle is 15 inches and the triangle's base is 6 inches. Monica says that the area of the fabric is 90 in². What error did Monica make? Explain your answer.

© Houghton Mifflin Harcourt Publishing Company



12. Multistep The sixth-grade art students are making a mosaic using tiles in the shape of right triangles. Each tile has leg measures of 3 centimeters and 5 centimeters. If there are 200 tiles in the mosaic, what is the area of the mosaic?

14.	Multistep Wayne is going to paint the side of the house shown in the diagram. What is the area that will be painted? Explain how you found your answer.	25 ft
H.	FOCUS ON HIGHER ORDER THINKING	Work Area
15.	Communicate Mathematical Ideas Explain how the areas of a triangle and a parallelogram with the same base and height are related.	
16.	Analyze Relationships A rectangle and a triangle have the same area. If their bases are the same lengths, how do their heights compare? Justify your answer.	
		urt Publishing Company
17.	What If? A right triangle has an area of 18 square inches.	in Harco
	 a. If the triangle is an isosceles triangle, what are the lengths of the legs of the triangle? 	© Houghton Miffi
	b. If the triangle is not an isosceles triangle, what are all the possible lengths of the legs, if the lengths are whole numbers?	

Solving Area 13.3 Equations



Find the area of right triangles, other triangles, special quadrilaterals, and polygons...; apply these techniques in the context of solving...problems. *Also 6.EE.7*

ESSENTIAL QUESTION

How do you use equations to solve problems about area of rectangles, parallelograms, trapezoids, and triangles?

Problem Solving Using the Area of a Triangle

Recall that the formula for the area of a triangle is $A = \frac{1}{2}bh$. You can also use the formula to find missing dimensions if you know the area and one dimension.

EXAMPLE 1 Real

The Hudson High School wrestling team just won the state tournament and has been awarded a triangular pennant to hang on the wall in the school gymnasium. The base of the pennant is 1.5 feet long. It has an area of 2.25 square feet. What is the height of the pennant?



COMMO

6.G.1, 6.EE.7

 $A = \frac{1}{2}bh$ Write the formula. $2.25 = \frac{1}{2}(1.5)h$ Use the formula to write an equation.2.25 = 0.75hMultiply $\frac{1}{2}$ and 1.5. $\frac{2.25}{0.75} = \frac{0.75h}{0.75}$ Divide both sides of the equation by 0.75.3 = h

The height of the pennant is 3 feet.

 Renee is sewing a quilt whose pattern contains right triangles. Each quilt piece has a height of 6 in. and an area of 24 in².

How long is the base of each quilt piece? _



the units for the height are in feet?

<u>Math</u> Talk

Mathematical Practices

How can you use units in

the formula to confirm that





Writing Equations Using the Area of a Trapezoid

You can use the formula for area of a trapezoid to write an equation to solve a problem.

Real **EXAMPLE 2**

A garden in the shape of a trapezoid has an area of 44.4 square meters. One base is 4.3 meters long and the other base is 10.5 meters long. The height of the trapezoid is the width of the garden. How wide is the garden?



COMMON CORE

6.G.1, 6.EE.7

4.3 m

Math Talk Mathematical Practices

How can you check that the answer is reasonable?

Divide both sides of the equation by 7.4.

The garden is 6 meters wide.

6 = h

Reflect

Communicate Mathematical Ideas Explain why the first step after 2. substituting is addition.





The cross section of a water bin is shaped like a trapezoid. The bases of the 3. trapezoid are 18 feet and 8 feet long. It has an area of 52 square feet. What is the height of the cross section?

Solving Multistep Problems

You can write and solve equations that represent real-world problems related to relationships in geometry.



John and Mary are using rolls of fabric to make a rectangular stage curtain for their class play. The rectangular piece of fabric on each roll measures 2.5 feet by 15 feet. If the area of the curtain is 200 square feet, what is the least number of rolls they need?

Analyze Information

Rewrite the question as a statement.

• Find the least number of rolls of fabric needed to cover an area of 200 ft².

List the important information.

- Each roll of fabric is a 2.5 foot by 15 foot rectangle.
- The area of the curtain is 200 square feet.

Formulate a Plan

Write an equation to find the area of each roll of fabric.

Use the area of the curtain and the area of each roll to write an equation to find the least number of rolls.

Solve

STEP 1

1) Write an equation to find the area of each roll of fabric.

$$A = lw$$
$$A = 15 \cdot 2.5$$
$$A = 37.5 \text{ ft}^2$$

STEP 2

STEP 3

ò

Write an equation to find the least number of rolls.

$$n = 200 \div 37.5$$

 $n = 5\frac{1}{3}$

The problem asks for the least number of rolls needed. Since 5 rolls will not be enough, they will need 6 rolls to make the curtain.

John and Mary will need 6 rolls of fabric to make the curtain.

် Justify and Evaluate

The area of each roll is about 38 ft². Since 38 ft² $\cdot\,6=228$ ft², the answer is reasonable.



6.G.1





4. A parallelogram-shaped field in a park needs sod. The parallelogram has a base of 21.5 meters and a height of 18 meters. The sod is sold in pallets of 50 square meters. How many pallets of sod are needed to fill the field?

Guided Practice

- **1.** A triangular bandana has an area of 70 square inches. The height of the triangle is $8\frac{3}{4}$ inches. Write and solve an equation to find the length of the base of the triangle. (Example 1)
- The top of a desk is shaped like a trapezoid. The bases of the trapezoid are 26.5 and 30 centimeters long. The area of the desk is 791 square centimeters. The height of the trapezoid is the width of the desk. Write and solve an equation to find the width of the desk. (Example 2)
- **3.** Taylor wants to paint his rectangular deck that is 42 feet long and 28 feet wide. A gallon of paint covers about 350 square feet. How many gallons of paint will Taylor need to cover the entire deck? (Example 3)

Write an equation to find the ______ of the deck.

Write and solve the equation.

Write an equation to find the ______.

Write and solve the equation.

Taylor will need _____ gallons of paint.

ESSENTIAL QUESTION CHECK-IN

4. How do you use equations to solve problems about area of rectangles, parallelograms, trapezoids, and triangles?

Class

Date

13.3 Independent Practice

COMMON 6.G.1, 6.EE.7

- **5.** A window shaped like a parallelogram has an area of $18\frac{1}{3}$ square feet. The height of the window is $3\frac{1}{3}$ feet. How long is the base of the window?
- **6.** A triangular sail has a base length of 2.5 meters. The area of the sail is 3.75 square meters. How tall is the sail?
- **7.** A section in a stained glass window is shaped like a trapezoid. The top base is 4 centimeters and the bottom base is 2.5 centimeters long. If the area of the section of glass is 3.9 square centimeters, how tall is the section?
- 8. Multistep Amelia wants to paint three walls in her family room. Two walls are 26 feet long by 9 feet wide. The other wall is 18 feet long by 9 feet wide.
 - a. What is the total area of the walls that Amelia wants to paint?
 - **b.** Each gallon of paint covers about 250 square feet. How many gallons of paint should Amelia buy to paint the walls?
- **9.** Critical Thinking The area of a triangular block is 64 square inches. If the base of the triangle is twice the height, how long are the base and the height of the triangle?
- **10. Multistep** Alex needs to varnish the top and the bottom of a dozen rectangular wooden planks. The planks are 8 feet long and 3 feet wide. Each pint of varnish covers about 125 square feet and costs \$3.50.
 - a. What is the total area that Alex needs to varnish?
 - **b.** How much will it cost Alex to varnish all the wooden planks?
- **11. Multistep** Leia cuts congruent triangular patches with an area of 45 square centimeters from a rectangular piece of fabric that is 18 centimeters long and 10 centimeters wide. How many of the patches can Leia cut from 32 pieces of the fabric?
- **12. Multistep** A farmer needs to buy fertilizer for two fields. One field is shaped like a trapezoid, and the other is shaped like a triangle. The trapezoidal field has bases that are 35 and 48 yards and a height of 26 yards. The triangular field has the same height as the trapezoidal field and a base of 39 yards. Each bag of fertilizer covers 150 square yards. How many bags of fertilizer does the farmer need to buy?





- **13.** A tennis court for singles play is 78 feet long and 27 feet wide.
 - **a.** The court for doubles play is 9 feet wider than the court for singles play. How much more area is covered by the tennis court used for doubles play?
 - **b.** The junior court for players 8 and under is 36 feet long and 18 feet wide. How much more area is covered by the tennis court used for singles play?
 - **c.** The court for players 10 and under is 18 feet shorter than the court for singles play. How much more area is covered by the tennis court used for singles play?
- 14. Draw Conclusions The cross section of a metal ingot is a trapezoid. The cross section has an area of 39 square centimeters. The top base of the cross section is 12 centimeters. The length of the bottom base is 2 centimeters greater than the top base. How tall is the metal ingot? Explain.

FOCUS ON HIGHER ORDER THINKING

- 15. Analyze Relationships A mirror is made of two congruent parallelograms as shown in the diagram. The parallelograms have a combined area of $9\frac{1}{3}$ square yards. The height of each parallelogram is $1\frac{1}{2}$ yards.
 - **a.** How long is the base of each parallelogram?
 - **b.** What is the area of the smallest *rectangle* of wall that the mirror could fit on?

16. Persevere in Problem Solving

A watercolor painting is 20 inches long by 9 inches wide. Ramon makes a border around the watercolor painting by making a mat that adds 1 inch to each side of the length and the width. What is the area of the mat?



Work Area



6.G.1 Find the area of ... polygons by composing into rectangles or decomposing into triangles and other shapes

ESSENTIAL QUESTION

How can you find the area of a polygon by breaking it into simpler shapes?

EXPLORE ACTIVITY

COMMON 6.G.1

Finding Areas Using Tangrams

A tangram is a square that is divided into smaller shapes. The area of the small square is 1 square unit. Use a tangram to find the area of each of the other tangram pieces.

A Place one large triangle on top of the other large triangle. What is true about these two triangles? What does this mean about the areas of these two triangles?



B Place the two small triangles on top of the square. What is the area of each small triangle? Write this area on the diagram.

C Arrange the square and one of the small triangles as shown.

What is the combined area?

Place the parallelogram and the other small triangle on top of the combined square and triangle. What is the area of the parallelogram? Explain.



Reflect

1. Critical Thinking Complete the rest of the diagram by filling in the remaining areas. Explain how you found your answers.



Finding Areas of Polygons

You can find the areas of polygons by breaking the polygons into smaller shapes. Then you can apply area formulas you already know.



2. Describe another way to find the area of the polygon in **B**.



Lesson 13.4 391





5. The diagram shows the floor plan of a hotel lobby. Carpet costs \$3 per square foot. How much will it cost to carpet the lobby?



Guided Practice

 In the diagram, the area of the large square is 1 square unit. Two diagonal segments divide the square into four equal-sized triangles. Two of these triangles are divided into smaller red and blue triangles that all have the same height and base length. Find the area of a red triangle. (Explore Activity)



Find the area of each polygon. (Example 1)



 Jess is painting a giant arrow on a playground. Find the area of the giant arrow. If one can of paint covers 100 square feet, how many cans should Jess buy? (Example 2)



ESSENTIAL QUESTION CHECK-IN

5. How can you find the area of a polygon that is not one for which you know an area formula?

C Houghton Mifflin Harcourt Publishing Company



- **9.** Multistep Cho is making banners shaped like triangles out of a rectangular piece of fabric. She cuts out two triangular banners as shown.
 - a. What is the area of a triangular banner?
 - **b.** What are the dimensions of the fabric left over after Cho cuts out the two banners?
 - **c.** What is the maximum number of banners that Cho can cut out from the fabric? Will she use all the fabric?



Work Area

H.O.T.

FOCUS ON HIGHER ORDER THINKING

- **10.** Persevere in Problem Solving The base of a parallelogram is 8 units, and the height is 5 units. A segment divides the parallelogram into two identical trapezoids. The height of each trapezoid is 5 units. Draw the parallelogram and the two trapezoids on the grid shown. Then find the area of one of the trapezoids.
- **11.** Persevere in Problem Solving The figure shown is a square with a triangular hole cut into one side. The ratio of the height *h* of the triangle to a side length of the square is 7 to 8. The ratio of the base *b* of the triangle to the side length of the square is 1 to 2. If the area of the square is 64 square inches, what is the area of the shaded part of the square? Show your work.



C Houghton Mifflin Harcourt Publishing Company

MODULE QUIZ





MODULE 13 MIXED REVIEW Assessment Readiness



Selected Response

1. The lengths of the diagonals of the rhombus are given. What is the area of the rhombus?



- A 161 in²
 C 644 in²
- **B** 322 in²
- (D) 966 in²
- 2. In the triangle below, the value of h is $\frac{3}{4}$ the side length that is labeled on the figure. What is the area of the triangle?



3. Tim is designing a logo. The logo is a polygon whose shape is a square attached to an equilateral triangle. The square and the equilateral triangle have side lengths of 2 centimeters, and the equilateral triangle has a height of about 1.7 cm. Find the area of the logo.

(C) 5.7 cm²

- (A) 1.7 cm²
- **(B)** 4 cm^2 **(D)** 7.4 cm^2

4. The trapezoid below has an area of 1,575 cm².



Which equation could you solve to find the height of the trapezoid?

(A) 45 <i>h</i> = 1,575	(C) 850.5 <i>h</i> = 1,575
(B) $90h = 1,575$	(D) 1,701 <i>h</i> = 1,575

Mini-Task

5. Cindy is designing a rectangular fountain in the middle of a courtyard. The rest of the courtyard will be covered in stone.



The part of the courtyard that will be covered in stone has an area of 246 square feet.

- **a.** What is the width of the fountain?
- **b.** What fraction of the area of the courtyard will be occupied by the fountain?