

# Factors and Multiples

MODULE



# 2



## ESSENTIAL QUESTION

How can you use greatest common factors and least common multiples to solve real-world problems?

LESSON 2.1

### Greatest Common Factor

COMMON CORE 6.NS.4

LESSON 2.2

### Least Common Multiple

COMMON CORE 6.NS.4



### Real-World Video

Organizers of banquets and other special events plan many things, including menus, seating arrangements, table decorations, and party favors. Factors and multiples can be helpful in this work.

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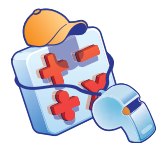
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# Are YOU Ready?

Complete these exercises to review skills you will need for this module.



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## Multiples

**EXAMPLE**  $5 \times 1 = 5$     $5 \times 2 = 10$     $5 \times 3 = 15$     $5 \times 4 = 20$     $5 \times 5 = 25$

To find the first five multiples of 5, multiply 5 by 1, 2, 3, 4, and 5.

List the first five multiples of the number.

1. 7 \_\_\_\_\_      2. 11 \_\_\_\_\_      3. 15 \_\_\_\_\_

## Factors

**EXAMPLE**  $1 \times 12 = 12$   
 $2 \times 6 = 12$   
 $3 \times 4 = 12$   
The factors of 12 are 1, 2, 3, 4, 6, 12.

To find the factors of 12, use multiplication facts of 12. Continue until pairs of factors repeat.

Write all the factors of the number.

4. 24 \_\_\_\_\_      5. 36 \_\_\_\_\_  
6. 45 \_\_\_\_\_      7. 32 \_\_\_\_\_

## Multiplication Properties (Distributive)

**EXAMPLE**  $7 \times 14 = 7 \times (10 + 4)$   
 $= (7 \times 10) + (7 \times 4)$   
 $= 70 + 28$   
 $= 98$

To multiply a number by a sum, multiply the number by each addend and add the products.

Use the Distributive Property to find the product.

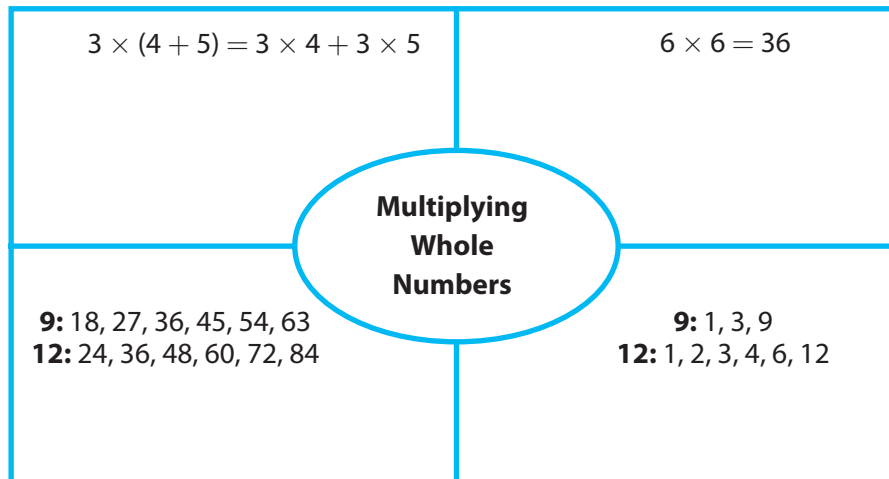
8.  $8 \times 15 = 8 \times (\square + \square)$   
 $= (\square \times \square) + (\square \times \square)$   
 $= \square + \square$   
 $= \square$

9.  $6 \times 17 = 6 \times (\square + \square)$   
 $= (\square \times \square) + (\square \times \square)$   
 $= \square + \square$   
 $= \square$

# Reading Start-Up

## Visualize Vocabulary

Use the ✓ words to complete the graphic.



## Vocabulary

### Review Words

- ✓ area (*área*)
- ✓ Distributive Property (*Propiedad distributiva*)
- ✓ factor (*factor*)
- ✓ multiple (*múltiplo*)
- ✓ product (*producto*)

### Preview Words

- greatest common factor (GCF) (*máximo común divisor (MCD)*)
- least common multiple (LCM) (*mínimo común múltiplo (m.c.m.)*)

## Understand Vocabulary

Complete the sentences below using the preview words.

1. Of all the whole numbers that divide evenly into two or more numbers, the one with the highest value is called the \_\_\_\_\_.
2. Of all the common products of two numbers, the one with the lowest value is called the \_\_\_\_\_.

## Active Reading

**Two-Panel Flip Chart** Create a two-panel flip chart to help you understand the concepts in this module. Label one flap "Greatest Common Factor." Label the other flap "Least Common Multiple." As you study each lesson, write important ideas under the appropriate flap.





# Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

**COMMON CORE** 6.NS.4

Find the **greatest common factor** of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the Distributive Property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

**Key Vocabulary**

**greatest common factor (GCF)**

*(máximo común divisor (MCD))*

The largest common factor of two or more given numbers.

## What It Means to You

You will determine the greatest common factor of two numbers and solve real-world problems involving the greatest common factor.

### UNPACKING EXAMPLE 6.NS.4

There are 12 boys and 18 girls in Ms. Ruiz’s science class. Each lab group must have the same number of boys and the same number of girls. What is the greatest number of groups Ms. Ruiz can make if every student must be in a group?

Factors of 12: 1, 2, 3, 4, 6, 12

Factors of 18: 1, 2, 3, 6, 9, 18

The GCF of 12 and 18 is 6. The greatest number of groups Ms. Ruiz can make is 6.

**COMMON CORE** 6.NS.4

Find the greatest common factor of two whole numbers less than or equal to 100 and the **least common multiple** of two whole numbers less than or equal to 12. ...

**Key Vocabulary**

**least common multiple (LCM)**

*(mínimo común múltiplo (m.c.m.))*

The smallest number, other than zero, that is a multiple of two or more given numbers.

## What It Means to You

You will determine the least common multiple of two numbers and solve real-world problems involving the least common multiple.

### UNPACKING EXAMPLE 6.NS.4

Lydia’s family will provide juice boxes and granola bars for 24 players. Juice comes in packs of 6, and granola bars in packs of 8. What is the least number of packs of each needed so that every player has a drink and a granola bar and there are none left over?



Multiples of 6: 6, 12, 18, 24, 30, ...

Multiples of 8: 8, 16, 24, 32, ...

The LCM of 6 and 8 is 24. Lydia’s family should buy  $24 \div 6 = 4$  packs of juice and  $24 \div 8 = 3$  packs of granola bars.



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# LESSON 2.1 Greatest Common Factor

COMMON CORE 6.NS.4

Find the greatest common factor of two whole numbers...



## ESSENTIAL QUESTION

How can you find and use the greatest common factor of two whole numbers?

### EXPLORE ACTIVITY 1



COMMON CORE 6.NS.4

## Understanding Common Factors

The **greatest common factor (GCF)** of two numbers is the greatest factor shared by those numbers.

**A florist makes bouquets from 18 roses and 30 tulips. All the bouquets will include both roses and tulips. If all the bouquets are identical, what are the possible bouquets that can be made?**



- A** Complete the tables to show the possible ways to divide each type of flower among the bouquets.

#### Roses

Number of Bouquets	1	2	3	6	9	18
Number of Roses in Each Bouquet	18	9				

#### Tulips

Number of Bouquets	1	2	3	5	6	10	15	30
Number of Tulips in Each Bouquet	30							

- B** Can the florist make five bouquets using all the flowers? Explain.  
\_\_\_\_\_
- C** What are the common factors of 18 and 30? What do they represent?  
\_\_\_\_\_
- D** What is the GCF of 18 and 30? \_\_\_\_\_

### Reflect

- 1. What If?** Suppose the florist has 18 roses and 36 tulips. What is the GCF of the numbers of roses and tulips? Explain.  
\_\_\_\_\_



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# Finding the Greatest Common Factor

One way to find the GCF of two numbers is to list all of their factors. Then you can identify common factors and the GCF.

## EXAMPLE 1



COMMON CORE

6.NS.4

A baker has 24 sesame bagels and 36 plain bagels to put into boxes. Each box must have the same number of each type of bagel. What is the greatest number of boxes that the baker can make using all of the bagels? How many sesame bagels and how many plain bagels will be in each box?



My Notes

The baker can divide 24 sesame bagels into groups of 1, 2, 3, 4, 6, 8, 12, or 24.

### STEP 1

List the factors of 24 and 36. Then circle the common factors.

Factors of 24: 1 2 3 4 6 8 12 24

Factors of 36: 1 2 3 4 6 9 12 18 36

### STEP 2

Find the GCF of 24 and 36.

The GCF is 12. So, the greatest number of boxes that the baker can make is 12. There will be 2 sesame bagels in each box, because  $24 \div 12 = 2$ . There will be 3 plain bagels, because  $36 \div 12 = 3$ .

## Reflect

2. **Critical Thinking** What is the GCF of two prime numbers? Give an example.

## YOUR TURN

Find the GCF of each pair of numbers.

3. 14 and 35 \_\_\_\_\_ 4. 20 and 28 \_\_\_\_\_

5. The sixth-grade class is competing in the school field day. There are 32 girls and 40 boys who want to participate. Each team must have the same number of girls and the same number of boys. What is the greatest number of teams that can be formed? How many boys and how many girls will be on each team?



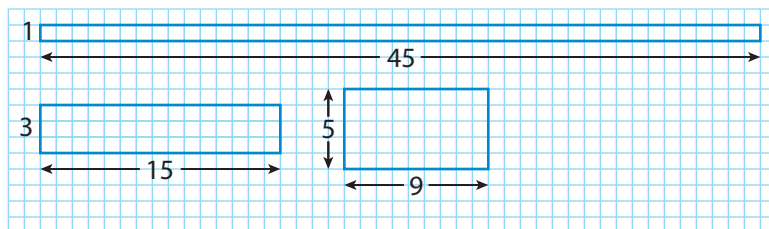
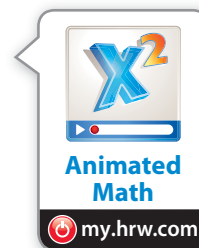
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# Using the Distributive Property

You can use the Distributive Property to rewrite a sum of two or more numbers as a product of their GCF and a sum of numbers with no common factor. To understand how, you can use grid paper to draw area models of 45 and 60. Here are all the possible area models of 45.



- A** What do the side lengths of the area models (1, 3, 5, 9, 15, and 45) represent? \_\_\_\_\_
- B** On your own grid paper, show all of the possible area models of 60.  
\_\_\_\_\_
- C** What side lengths do the area models of 45 and 60 have in common? What do the side lengths represent?  
\_\_\_\_\_
- D** What is the greatest common side length? What does it represent?  
\_\_\_\_\_
- E** Write 45 as a product of the GCF and another number. \_\_\_\_\_  
Write 60 as a product of the GCF and another number. \_\_\_\_\_
- F** Use your answers above to rewrite  $45 + 60$ .

$$45 + 60 = 15 \times \underline{\quad} + 15 \times \underline{\quad}$$

Use the Distributive Property and your answer above to write  $45 + 60$  as a product of the GCF and a sum of two numbers.

$$15 \times \underline{\quad} + 15 \times \underline{\quad} = 15 \times (\underline{\quad} + \underline{\quad}) = 15 \times 7$$

## Math Talk

### Mathematical Practices

How can you check to see if your product is correct?

## Reflect

**Write the sum of the numbers as the product of their GCF and another sum.**

**6.**  $27 + 18$  \_\_\_\_\_      **7.**  $120 + 36$  \_\_\_\_\_

**8.**  $9 + 35$  \_\_\_\_\_

## Guided Practice

1. Lee is sewing vests using 16 green buttons and 24 blue buttons. All the vests are identical, and all have both green and blue buttons. What are the possible numbers of vests Lee can make? What is the greatest number of vests Lee can make? (*Explore Activity 1, Example 1*)

List the factors of 16 and 24. Then circle the common factors.

<b>Factors of 16:</b>								
<b>Factors of 24:</b>								

What are the common factors of 16 and 24? \_\_\_\_\_

What are the possible numbers of vests Lee can make? \_\_\_\_\_

What is the GCF of 16 and 24? \_\_\_\_\_

What is the greatest number of vests Lee can make? \_\_\_\_\_

**Write the sum of numbers as a product of their GCF and another sum.**

(*Explore Activity 2*)

2.  $36 + 45$

What is the GCF of 36 and 45? \_\_\_\_\_

Write each number as a product of the GCF and another number.

Then use the Distributive Property to rewrite the sum.

$$\left(\square \times \square\right) + \left(\square \times \square\right) = \left(\square\right) \times \left(\square + \square\right)$$

3.  $75 + 90$

What is the GCF of 75 and 90? \_\_\_\_\_

Write each number as a product of the GCF and another number.

Then use the Distributive Property to rewrite the sum.

$$\left(\square \times \square\right) + \left(\square \times \square\right) = \left(\square\right) \times \left(\square + \square\right)$$



### ESSENTIAL QUESTION CHECK-IN

4. Describe how to find the GCF of two numbers.

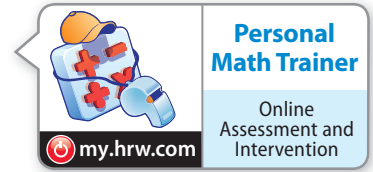
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# 2.1 Independent Practice

**COMMON CORE** 6.NS.4



**List the factors of each number.**

5. 12 \_\_\_\_\_

6. 50 \_\_\_\_\_

7. 39 \_\_\_\_\_

8. 64 \_\_\_\_\_

**Find the GCF of each pair of numbers.**

9. 40 and 48 \_\_\_\_\_

10. 30 and 45 \_\_\_\_\_

11. 10 and 45 \_\_\_\_\_

12. 25 and 90 \_\_\_\_\_

13. 21 and 40 \_\_\_\_\_

14. 28 and 70 \_\_\_\_\_

15. 60 and 72 \_\_\_\_\_

16. 45 and 81 \_\_\_\_\_

17. 28 and 32 \_\_\_\_\_

18. 55 and 77 \_\_\_\_\_

19. Carlos is arranging books on shelves. He has 24 novels and 16 autobiographies. Each shelf will have the same numbers of novels and autobiographies. If Carlos must place all of the books on shelves, what are the possible numbers of shelves Carlos will use?

\_\_\_\_\_

20. The middle school band has 56 members. The high school band has 96 members. The bands are going to march one after the other in a parade. The director wants to arrange the bands into the same number of columns. What is the greatest number of columns in which the two bands can be arranged if each column has the same number of marchers? How many band members will be in each column?

\_\_\_\_\_

21. For football tryouts at a local school, 12 coaches and 42 players will split into groups. Each group will have the same numbers of coaches and players. What is the greatest number of groups that can be formed? How many coaches and players will be in each of these groups?

\_\_\_\_\_

22. Lola is placing appetizers on plates. She has 63 spring rolls and 84 cheese cubes. She wants to include both appetizers on each plate. Each plate must have the same numbers of spring rolls and cheese cubes. What is the greatest number of plates she can make using all of the appetizers? How many of each type of appetizer will be on each of these plates?

\_\_\_\_\_



Write the sum of the numbers as the product of their GCF and another sum.

23.  $56 + 64$  \_\_\_\_\_      24.  $48 + 14$  \_\_\_\_\_  
25.  $30 + 54$  \_\_\_\_\_      26.  $24 + 40$  \_\_\_\_\_  
27.  $55 + 66$  \_\_\_\_\_      28.  $49 + 63$  \_\_\_\_\_  
29.  $40 + 25$  \_\_\_\_\_      30.  $63 + 15$  \_\_\_\_\_

31. **Vocabulary** Explain why the greatest common factor of two numbers is sometimes 1.

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**FOCUS ON HIGHER ORDER THINKING**

32. **Communicate Mathematical Ideas** Tasha believes that she can rewrite the difference  $120 - 36$  as a product of the GCF of the two numbers and another difference. Is she correct? Explain your answer.

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33. **Persevere in Problem Solving** Explain how to find the greatest common factor of three numbers.

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34. **Critique Reasoning** Xiao's teacher asked him to rewrite the sum  $60 + 90$  as the product of the GCF of the two numbers and a sum. Xiao wrote  $3(20 + 30)$ . What mistake did Xiao make? How should he have written the sum?

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Work Area

# LESSON 2.2 Least Common Multiple

COMMON CORE 6.NS.4

Find ... the least common multiple of two whole numbers...



## ESSENTIAL QUESTION

How do you find and use the least common multiple of two numbers?

### EXPLORE ACTIVITY



COMMON CORE 6.NS.4

## Finding the Least Common Multiple

A multiple of a number is the product of the number and another number. For example, 9 is a multiple of the number 3. The **least common multiple (LCM)** of two or more numbers is the least number, other than zero, that is a multiple of all the numbers.

**Ned is training for a triathlon. He will swim every sixth day and bicycle every eighth day. On what days will he both swim and bicycle?**



- A** In the chart below, shade each day that Ned will swim. Circle each day Ned will bicycle.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- B** On what days will Ned both swim and bicycle?

The numbers of the days that Ned will swim and bicycle are common multiples of 6 and 8.

### Reflect

1. **Interpret the Answer** What does the LCM represent in this situation?

\_\_\_\_\_



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# Applying the LCM

You can use the LCM of two whole numbers to solve problems.

## EXAMPLE 1



COMMON CORE 6.NS.4

A store is holding a promotion. Every third customer receives a free key chain, and every fourth customer receives a free magnet. Which customer will be the first to receive both a key chain and a magnet?

**STEP 1** List the multiples of 3 and 4. Then circle the common multiples.

**Multiples of 3:** 3   6   9   12   15   18   21   24   27

**Multiples of 4:** 4   8   12   16   20   24   28   32   36

**STEP 2** Find the LCM of 3 and 4.

The LCM is 12.

The first customer to get both a key chain and a magnet is the 12th customer.

### Math Talk

Mathematical Practices

What steps do you take to list the multiples of a number?

### YOUR TURN

2. Find the LCM of 4 and 9 by listing the multiples. \_\_\_\_\_

**Multiples of 4:** \_\_\_\_\_

**Multiples of 9:** \_\_\_\_\_



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## Guided Practice

1. After every ninth visit to a restaurant you receive a free beverage. After every twelfth visit you receive a free appetizer. If you visit the restaurant 100 times, on which visits will you receive a free beverage and a free appetizer? At which visit will you first receive a free beverage and a free appetizer? (Explore Activity 1, Example 1)

\_\_\_\_\_



### ESSENTIAL QUESTION CHECK-IN


2. What steps can you take to find the LCM of two numbers?

\_\_\_\_\_

\_\_\_\_\_

# 2.2 Independent Practice

**COMMON CORE** 6.NS.4



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**Find the LCM of each pair of numbers.**

- 3. 8 and 56 \_\_\_\_\_
- 4. 25 and 50 \_\_\_\_\_
- 5. 12 and 30 \_\_\_\_\_
- 6. 6 and 10 \_\_\_\_\_
- 7. 16 and 24 \_\_\_\_\_
- 8. 14 and 21 \_\_\_\_\_
- 9. 9 and 15 \_\_\_\_\_
- 10. 5 and 11 \_\_\_\_\_

11. During February, Kevin will water his ivy every third day, and water his cactus every fifth day.

a. On which date will Kevin first water both plants together?

\_\_\_\_\_

b. Will Kevin water both plants together again in February? Explain.

\_\_\_\_\_

\_\_\_\_\_

12. **Vocabulary** Given any two numbers, which is greater, the LCM of the numbers or the GCF of the numbers? Why?

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

**Use the subway train schedule.**

13. The red line and the blue line trains just arrived at the station. When will they next arrive at the station at the same time?

In \_\_\_\_\_ minutes

14. The blue line and the yellow line trains just arrived at the station. When will they next arrive at the station at the same time?

In \_\_\_\_\_ minutes

15. All three trains just arrived at the station. When will they next all arrive at the station at the same time?

In \_\_\_\_\_ minutes



Train Schedule	
Train	Arrives Every...
Red line	8 minutes
Blue line	10 minutes
Yellow line	12 minutes

16. You buy a lily and an African violet on the same day. You are instructed to water the lily every fourth day and water the violet every seventh day after taking them home. What is the first day on which you will water both plants on the same day? How can you use this answer to determine each of the next days you will water both plants on the same day?

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**FOCUS ON HIGHER ORDER THINKING**

17. What is the LCM of two numbers if one number is a multiple of the other? Give an example.

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18. What is the LCM of two numbers that have no common factors greater than 1? Give an example.

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19. **Draw Conclusions** The least common multiple of two numbers is 60, and one of the numbers is 7 less than the other number. What are the numbers? Justify your answer.

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20. **Communicate Mathematical Ideas** Describe how to find the least common multiple of three numbers. Give an example.

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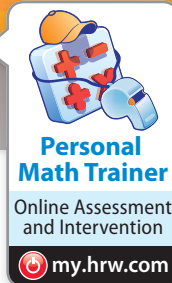
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Work Area

# Ready to Go On?



## 2.1 Greatest Common Factor

Find the GCF of each pair of numbers.

- 1. 20 and 32 \_\_\_\_\_
- 2. 24 and 56 \_\_\_\_\_
- 3. 36 and 90 \_\_\_\_\_
- 4. 45 and 75 \_\_\_\_\_
- 5. 28 girls and 32 boys volunteer to plant trees at a school. The principal divides the girls and boys into identical groups that have girls and boys in each group. What is the greatest number of groups the principal can make? \_\_\_\_\_

Write the sum of the numbers as the product of their GCF and another sum.

- 6.  $32 + 20$  \_\_\_\_\_
- 7.  $18 + 27$  \_\_\_\_\_

## 2.2 Least Common Multiple

Find the LCM of each pair of numbers.

- 8. 6 and 12 \_\_\_\_\_
- 9. 6 and 10 \_\_\_\_\_
- 10. 8 and 9 \_\_\_\_\_
- 11. 9 and 12 \_\_\_\_\_
- 12. Juanita runs every third day and swims every fifth day. If Juanita runs and swims today, in how many days will she run and swim again on the same day? \_\_\_\_\_



### ESSENTIAL QUESTION

- 13. What types of problems can be solved using the greatest common factor? What types of problems can be solved using the least common multiple?

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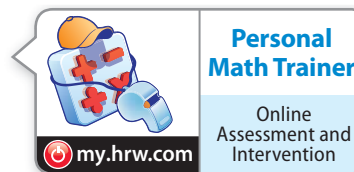
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# Assessment Readiness



## Selected Response

- What is the least common multiple of 5 and 150?  
 (A) 5                      (C) 15  
 (B) 50                    (D) 150
- Cy has 42 baseball cards and 70 football cards that he wants to group into packages. Each package will have the same number of cards, and each package will have the same numbers of baseball cards and football cards. How many packages will Cy make if he uses all of the cards?  
 (A) 7                      (C) 14  
 (B) 10                    (D) 21
- During a promotional event, a sporting goods store gave a free T-shirt to every 8th customer and a free water bottle to every 10th customer. Which customer was the first to get a free T-shirt and a free water bottle?  
 (A) the 10th customer  
 (B) the 20th customer  
 (C) the 40th customer  
 (D) the 80th customer

- The table below shows the positions relative to sea level of four divers.

Kareem	Li	Maria	Tara
-8 ft	-10 ft	-9 ft	-7 ft

Which diver is farthest from the surface?

- (A) Kareem                      (C) Maria  
 (B) Li                              (D) Tara

- What is the greatest common factor of 12 and 16?  
 (A) 2                      (C) 12  
 (B) 4                      (D) 48
- Which expression is equivalent to  $27 + 15$ ?  
 (A)  $9 \times (3 + 5)$   
 (B)  $3 \times (9 + 15)$   
 (C)  $9 \times (3 + 15)$   
 (D)  $3 \times (9 + 5)$
- During a science experiment, the temperature of a solution in Beaker 1 was 5 degrees below zero. The temperature of a solution in Beaker 2 was the opposite of the temperature in Beaker 1. What was the temperature in Beaker 2?  
 (A) -5 degrees              (C) 5 degrees  
 (B) 0 degrees                (D) 10 degrees

## Mini-Task

- Tia is buying paper cups and plates. Cups come in packages of 12, and plates come in packages of 10. She wants to buy the same number of cups and plates, but plans to buy the least number of packages possible. How much should Tia expect to pay if each package of cups is \$3 and each package of plates is \$5? Explain.

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