Applying Ratios and Rates

MODULE 7

COMMON CORE

ESSENTIAL QUESTION

How can you use ratios and rates to solve real-world problems?

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Real-World Video

Chefs use lots of measurements when preparing meals. If a chef needs more or less of a dish, he can use ratios to scale the recipe up or down. Using proportional reasoning, the chef keeps the ratios of all ingredients constant.

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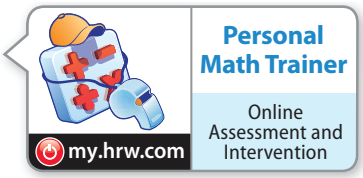
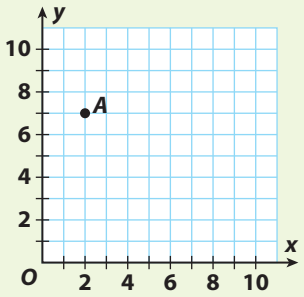
Are YOU Ready?

Complete these exercises to review skills you will need for this module.



Graph Ordered Pairs (First Quadrant)

EXAMPLE



To graph $A(2, 7)$, start at the origin.
Move 2 units right.
Then move 7 units up.
Graph point $A(2, 7)$.

Graph each ordered pair on the coordinate plane above.

1. $B(9, 6)$ **2.** $C(0, 2)$ **3.** $D(6, 10)$ **4.** $E(3, 4)$

Write Equivalent Fractions

EXAMPLE

$\frac{14}{21} = \frac{14 \times 2}{21 \times 2} = \frac{28}{42}$

$\frac{14}{21} = \frac{14 \div 7}{21 \div 7} = \frac{2}{3}$

Multiply the numerator and denominator by the same number to find an equivalent fraction.
Divide the numerator and denominator by the same number to find an equivalent fraction.

Write the equivalent fraction.

5. $\frac{6}{8} = \frac{\square}{32}$ **6.** $\frac{4}{6} = \frac{\square}{12}$ **7.** $\frac{1}{8} = \frac{\square}{56}$ **8.** $\frac{9}{12} = \frac{\square}{4}$

9. $\frac{5}{9} = \frac{25}{\square}$ **10.** $\frac{5}{6} = \frac{20}{\square}$ **11.** $\frac{36}{45} = \frac{12}{\square}$ **12.** $\frac{20}{36} = \frac{10}{\square}$

Multiples

EXAMPLE List the first five multiples of 4.

$4 \times 1 = 4$
$4 \times 2 = 8$
$4 \times 3 = 12$
$4 \times 4 = 16$
$4 \times 5 = 20$

Multiply 4 by the numbers 1, 2, 3, 4, and 5.

List the first five multiples of each number.

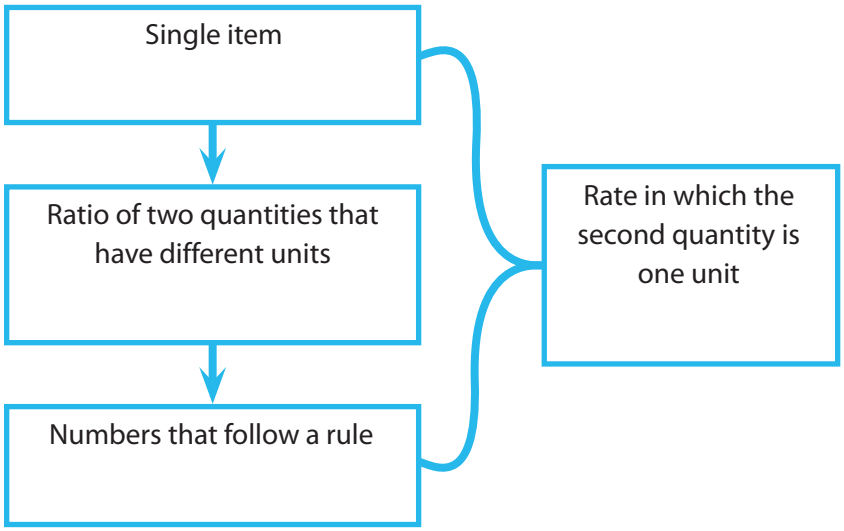
13. 3 _____ **14.** 7 _____ **15.** 8 _____

Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the graphic.

Comparing Unit Rates



Understand Vocabulary

Complete the sentences using the preview words.

1. A _____ is a rate that compares two equivalent measurements.

2. In a scale drawing, the _____ describes how the dimensions in the actual object compare to the dimensions in the drawing.

Vocabulary

Review Words

- equivalent ratios *(razones equivalentes)*
- factor *(factor)*
- graph *(gráfica)*
- ✔ pattern *(patrón)*
- point *(punto)*
- ✔ rate *(tasa)*
- ratio *(razón)*
- ✔ unit *(unidad)*
- ✔ unit rate *(tasa unitaria)*

Preview Words

- conversion factor *(factor de conversión)*
- proportion *(proporción)*
- scale *(escala)*
- scale drawing *(dibujo a escala)*

Active Reading

Tri-Fold Before beginning the module, create a tri-fold to help you learn the concepts and vocabulary in this module. Fold the paper into three sections. Label one column "Rates and Ratios," the second column "Proportions," and the third column "Converting Measurements." Complete the tri-fold with important vocabulary, examples, and notes as you read the module.

MODULE 7

Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

COMMON CORE 6.RP.3

Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Key Vocabulary

ratio *(razón)*
A comparison of two quantities by division.

rate *(tasa)*
A ratio that compares two quantities measured in different units.

What It Means to You

You will use ratios and rates to solve real-world problems such as those involving proportions.

UNPACKING EXAMPLE 6.RP.3

The distance from Austin to Dallas is about 200 miles. How far apart will these cities appear on a map with the scale of $\frac{1 \text{ in.}}{50 \text{ mi}}$?

$$\frac{1 \text{ inch}}{50 \text{ miles}} = \frac{\square \text{ inches}}{200 \text{ miles}}$$ *Write the scale as a unit rate.*

$$\frac{1 \text{ inch} \times 4}{50 \text{ miles} \times 4} = \frac{\square \text{ inches}}{200 \text{ miles}}$$ *200 is a common denominator.*

$$\square = 4$$

Austin and Dallas are 4 inches apart on the map.

COMMON CORE 6.RP.3d

Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Key Vocabulary

unit rate *(tasa unitaria)*
A rate in which the second quantity in the comparison is one unit.

What It Means to You

You will use unit rates to convert measurement units.

UNPACKING EXAMPLE 6.RP.3d

The Washington Monument is about 185 yards tall. This height is almost equal to the length of two football fields. About how many feet is this?

$$185 \text{ yd} \cdot \frac{3 \text{ ft}}{1 \text{ yd}}$$

$$= \frac{185 \text{ yd}}{1} \cdot \frac{3 \text{ ft}}{1 \text{ yd}}$$

$$= 555 \text{ ft}$$

The Washington Monument is about 555 feet tall.

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LESSON 7.1

Ratios, Rates, Tables, and Graphs

COMMON CORE 6.RP.3a

Make tables of equivalent ratios ..., find missing values in the tables, and plot the pairs of values on the coordinate plane. ... *Also 6.RP.3, 6.RP.3b*

ESSENTIAL QUESTION

How can you represent real-world problems involving ratios and rates with tables and graphs?

EXPLORE ACTIVITY 1 Real World

COMMON CORE 6.RP.3, 6.RP.3a

Finding Ratios from Tables

Students in Mr. Webster's science classes are doing an experiment that requires 250 milliliters of distilled water for every 5 milliliters of ammonia. The table shows the amount of distilled water needed for various amounts of ammonia.

Ammonia (mL)	2	3	3.5		5
Distilled water (mL)	100			200	250

A Use the numbers in the first column of the table to write a ratio of distilled water to ammonia. _____

B How much distilled water is used for 1 milliliter of ammonia? _____

Use your answer to write another ratio of distilled water to ammonia.

_____

C The ratios in A and B are **equivalent/not equivalent.**

D How can you use your answer to B to find the amount of distilled water to add to a given amount of ammonia?

_____

_____

Math Talk

Mathematical Practices

Is the relationship between the amount of ammonia and the amount of distilled water additive or multiplicative? Explain.

E Complete the table. What are the equivalent ratios shown in the table?

$$\frac{100}{2} = \frac{\square}{3} = \frac{\square}{3.5} = \frac{200}{\square} = \frac{250}{5}$$

Reflect

1. **Look for a Pattern** When the amount of ammonia increases by 1 milliliter, the amount of distilled water increases by _____ milliliters. So 6 milliliters of ammonia requires _____ milliliters of distilled water.

EXPLORE ACTIVITY 2 Real World

COMMON CORE 6.RP.3a

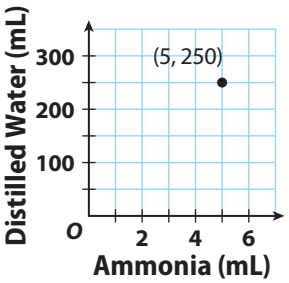
Graphing with Ratios

A Copy the table from Explore Activity 1 that shows the amounts of ammonia and distilled water.

Ammonia (mL)	2	3	3.5		5
Distilled water (mL)	100			200	250

B Write the information in the table as ordered pairs. Use the amount of ammonia as the *x*-coordinates and the amount of distilled water as the *y*-coordinates.

(2, _____) (3, _____), (3.5, _____), (_____, 200), (5, 250)



Graph the ordered pairs. Because fractions and decimals can represent amounts of chemicals, connect the points.

Describe your graph. _____

C For each ordered pair that you graphed, write the ratio of the *y*-coordinate to the *x*-coordinate. _____

D The ratio of distilled water to ammonia is $\frac{\square}{1}$. How are the ratios in **C** related to this ratio? _____

E The point (2.5, 125) is on the graph but not in the table. The ratio of the *y*-coordinate to the *x*-coordinate is _____. How is this ratio related to the ratios in **C** and **D**? _____

2.5 milliliters of ammonia requires _____ milliliters of distilled water.

F **Conjecture** What do you think is true for every point on the graph?

_____

_____

Reflect

2. **Communicate Mathematical Ideas** How can you use the graph to find the amount of distilled water to use for 4.5 milliliters of ammonia?

_____

_____

Representing Rates with Tables and Graphs

You can use tables and graphs to represent real-world problems involving equivalent rates.

EXAMPLE 1

Real World

COMMON CORE 6.RP.3a, 6.RP.3b

The Webster family is taking an express train to Washington, D.C. The train travels at a constant speed and makes the trip in 2 hours.

A Make a table to show the distance the train travels in various amounts of time.

STEP 1 Write a ratio of distance to time to find the rate.

$$\frac{\text{distance}}{\text{time}} = \frac{120 \text{ miles}}{2 \text{ hours}} = \frac{60 \text{ miles}}{1 \text{ hour}} = 60 \text{ miles per hour}$$

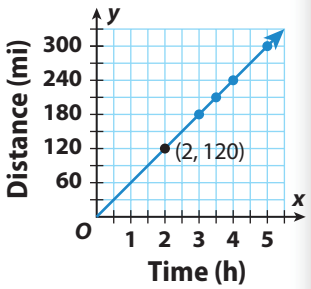
STEP 2 Use the unit rate to make a table.

Time (h)	2	3	3.5	4	5
Distance (mi)	120	180	210	240	300

B Graph the information from the table.

STEP 1 Write ordered pairs. Use Time as the *x*-coordinates and Distance as the *y*-coordinates.

(2, 120), (3, 180), (3.5, 210), (4, 240), (5, 300)

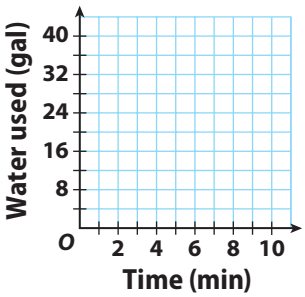


STEP 2 Graph the ordered pairs and connect the points.

YOUR TURN

3. A shower uses 12 gallons of water in 3 minutes. Complete the table and graph.

Time (min)	2	3	3.5		6.5
Water used (gal)				20	



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Guided Practice

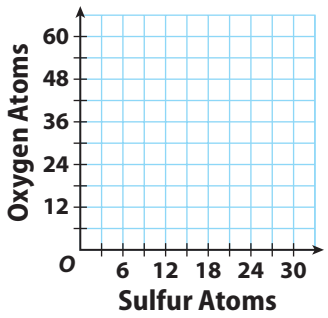
1. The ratio of oxygen atoms to sulfur atoms in sulfur dioxide is always the same. The table shows the numbers of atoms in different quantities of sulfur dioxide. Complete the table. (Explore Activity 1)

Sulfur atoms	6	9	21	
Oxygen atoms	12			54

What are the equivalent ratios shown in the table?

_____

2. Use the table in Exercise 1 to graph the relationship between sulfur atoms and oxygen atoms. (Explore Activity 2)



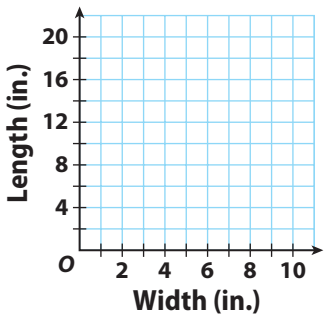
3. Stickers are made with the same ratio of width to length. A sticker 2 inches wide has a length of 4 inches. Complete the table. (Explore Activity 1)

Width (in.)	2	4	7	
Length (in.)				16

What are the equivalent ratios shown in the table?

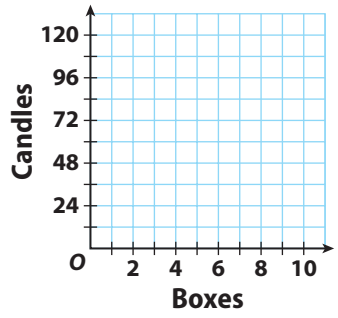
_____

4. Graph the relationship between the width and the length of the stickers from Exercise 3. (Explore Activity 2)



5. Five boxes of candles contain a total of 60 candles. Each box holds the same number of candles. Complete the table and graph the relationship. (Example 1)

Boxes	5	8	
Candles			120



ESSENTIAL QUESTION CHECK-IN

6. How do you represent real-world problems involving ratios and rates with tables and graphs?

_____

_____

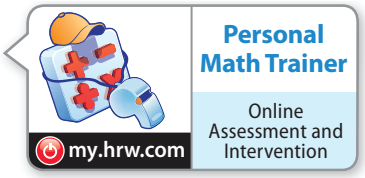
_____

_____

Name _____ Class _____ Date _____

7.1 Independent Practice

COMMON CORE 6.RP.3, 6.RP.3a, 6.RP.3b



The table shows information about the number of sweatshirts sold and the money collected at a fundraiser for school athletic programs. For Exercises 7–12, use the table.

Sweatshirts sold	3	5	8		12
Money collected ($)	60			180	

7. Find the rate of money collected per sweatshirt sold. Show your work.

_____

_____

8. Use the unit rate to complete the table.

9. Explain how to graph information from the table.

_____

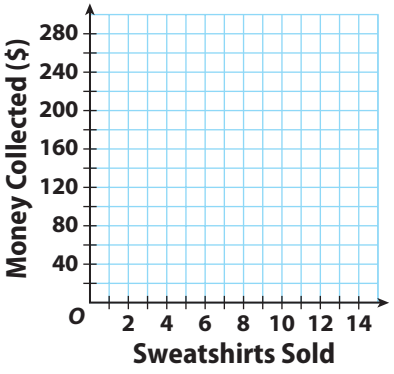
_____

_____

_____

10. Write the information in the table as ordered pairs. Graph the relationship from the table.

_____



11. **What If?** How much money would be collected if 24 sweatshirts were sold? Show your work.

_____

_____

_____

12. **Analyze Relationships** Does the point (5.5, 110) make sense in this context? Explain.

_____

_____

_____

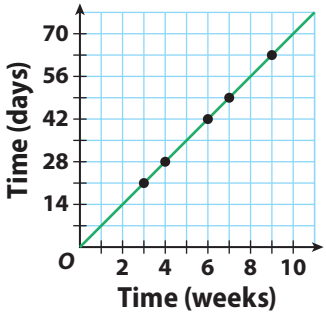
13. **Communicate Mathematical Ideas** The table shows the distance Randy drove on one day of her vacation. Find the distance Randy would have gone if she had driven for one more hour at the same rate. Explain how you solved the problem.

Time (h)	1	2	3	4	5
Distance (mi)	55	110	165	220	275

_____

_____

Use the graph for Exercises 14–15.



14. **Analyze Relationships** How many weeks correspond to 56 days? Explain.

_____

15. **Represent Real-World Problems** What is a real-life relationship that might be described by the graph?

_____

_____

H.O.T. FOCUS ON HIGHER ORDER THINKING

16. **Make a Conjecture** Complete the table. Then find the rates $\frac{\text{distance}}{\text{time}}$ and $\frac{\text{time}}{\text{distance}}$.

Time (min)	1	2	5	
Distance (m)			25	100

$\frac{\text{distance}}{\text{time}} =$ _____

$\frac{\text{time}}{\text{distance}} =$ _____

a. Are the $\frac{\text{time}}{\text{distance}}$ rates equivalent? Explain.

_____

b. Suppose you graph the points (time, distance) and your friend graphs (distance, time). How will your graphs be different?

_____

_____

_____

17. **Communicate Mathematical Ideas** To graph a rate or ratio from a table, how do you determine the scales to use on each axis?

_____

_____

_____

Work Area

LESSON 7.2

Solving Problems with Proportions

COMMON CORE 6.RP.3

Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about… equations. *Also 6.RP.3b*

ESSENTIAL QUESTION

How can you solve problems with proportions?

Using Equivalent Ratios to Solve Proportions

A **proportion** is a statement that two ratios or rates are equivalent.

$\frac{1}{3}$ and $\frac{2}{6}$ are equivalent ratios. $\frac{1}{3} = \frac{2}{6}$ is a proportion.

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EXAMPLE 1

Real World

COMMON CORE 6.RP.3

Sheldon and Leonard are partners in a business. Sheldon makes $2 in profits for every $5 that Leonard makes. If Leonard makes $20 profit on the first item they sell, how much profit does Sheldon make?

STEP 1 Write a proportion.

Sheldon's profit is unknown.

$$\frac{\text{Sheldon's profit}}{\text{Leonard's profit}} \rightarrow \frac{\$2}{\$5} = \frac{\square}{\$20} \leftarrow \frac{\text{Sheldon's profit}}{\text{Leonard's profit}}$$

STEP 2 Use common denominators to write equivalent ratios.

$$\frac{\$2 \times 4}{\$5 \times 4} = \frac{\square}{\$20}$$

20 is a common denominator.

$$\frac{\$8}{\$20} = \frac{\square}{\$20}$$

Equivalent ratios with the same denominators have the same numerators.

$$\square = \$8$$

If Leonard makes $20 profit, Sheldon makes $8 profit.

Math Talk
Mathematical Practices

For every dollar that Leonard makes, how much does Sheldon make? Explain.

YOUR TURN

1. The members of the PTA are ordering pizza for a meeting. They plan to order 2 cheese pizzas for every 3 pepperoni pizzas they order. How many cheese pizzas will they order if they order 15 pepperoni pizzas?

_____

_____

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Using Unit Rates to Solve Proportions

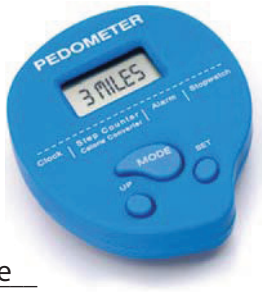
You can also use equivalent rates to solve proportions. Finding a unit rate may help you write equivalent rates.

EXAMPLE 2

Real World

COMMON CORE 6.RP.3b

The distance Ali runs in 36 minutes is shown on the pedometer. At this rate, how far could he run in 60 minutes?



STEP 1 Write a proportion.

$$\frac{\text{time}}{\text{distance}} \rightarrow \frac{36 \text{ minutes}}{3 \text{ miles}} = \frac{60 \text{ minutes}}{\square \text{ miles}} \leftarrow \frac{\text{time}}{\text{distance}}$$

60 is not a multiple of 36. So, there is no whole number by which you can multiply 3 miles to find $\square$.

STEP 2 Find the unit rate of the rate you know.

You know that Ali runs 3 miles in 36 minutes.

$$\frac{36 \div 3}{3 \div 3} = \frac{12}{1}$$

The unit rate is 12 minutes per 1 mile.

STEP 3 Use the unit rate to write an equivalent rate that compares 60 miles to an unknown number of minutes.

Think: You can multiply $12 \times 5 = 60$. So multiply the denominator by the same number.

$$\frac{12 \times 5}{1 \times 5} = \frac{60}{\square}$$

$$\frac{60}{5} = \frac{60}{\square}$$

$\square = 5$ miles

Equivalent rates with the same numerators have the same denominators.

At this rate, Ali can run 5 miles in 60 minutes.

Math Talk
Mathematical Practices

Compare the fractions $\frac{36}{3}$ and $\frac{60}{5}$ using $<$, $>$ or $=$. Explain.

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YOUR TURN

2. Ms. Reynold's sprinkler system has 9 stations that water all the parts of her front and back lawn. Each station runs for an equal amount of time. If it takes 48 minutes for the first 4 stations to water, how long does it take to water all parts of her lawn? _____

Using Proportional Relationships to Find Distance on a Map

A **scale drawing** is a drawing of a real object that is proportionally smaller or larger than the real object. A **scale** describes how the dimensions in the objects compare.

A map is a scale drawing. The measurements on a map are in proportion to the actual distances. If 1 inch on a map equals an actual distance of 2 miles, the scale is 1 inch = 2 miles. You can write a scale as a rate to solve problems.

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EXAMPLE 3

Real World

COMMON CORE 6.RP.3b

The distance between two schools on Lehigh Avenue is shown on the map. What is the actual distance between the schools?

Scale: 1 inch = 2 miles

STEP 1 Write a proportion.

$$\frac{2 \text{ miles}}{1 \text{ inch}} = \frac{\square \text{ miles}}{3 \text{ inches}}$$

Write the scale as a unit rate.

STEP 2 Write an equivalent rate to find the missing number.

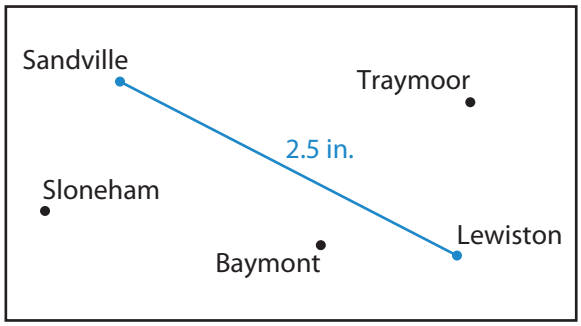
$$\frac{2 \text{ miles} \times 3}{1 \text{ inch} \times 3} = \frac{6 \text{ miles}}{3 \text{ inches}}$$

So, in Step 1, the missing number is 6.

The actual distance between the two schools is 6 miles.

YOUR TURN

3. The distance between Sandville and Lewiston is shown on the map. What is the actual distance between the towns?



Scale: 1 inch = 20 miles

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Guided Practice

Find the unknown value in each proportion. (Example 1)

1. $\frac{3}{5} = \frac{\square}{30}$

$\frac{3 \times \square}{5 \times \square} = \frac{\square}{30}$

2. $\frac{4}{10} = \frac{\square}{5}$

$\frac{4 \div \square}{10 \div \square} = \frac{\square}{5}$

Solve using equivalent ratios. (Example 1)

3. Leila and Jo are two of the partners in a business. Leila makes $3 in profits for every $4 that Jo makes. If Jo makes $60 profit on the first item they sell, how much profit does Leila make? _____

4. Hendrick wants to enlarge a photo that is 4 inches wide and 6 inches tall. The enlarged photo keeps the same ratio. How tall is the enlarged photo if it is 12 inches wide? _____

Solve using unit rates. (Example 2)

5. A person on a moving sidewalk travels 21 feet in 7 seconds. The moving sidewalk has a length of 180 feet. How long will it take to move from one end of the sidewalk to the other?

_____

6. In a repeating musical pattern, there are 56 beats in 7 measures. How many measures are there in 104 beats?

_____

7. Contestants in a dance-a-thon rest for the same amount of time every hour. A couple rests for 25 minutes in 5 hours. How long did they rest in 8 hours?

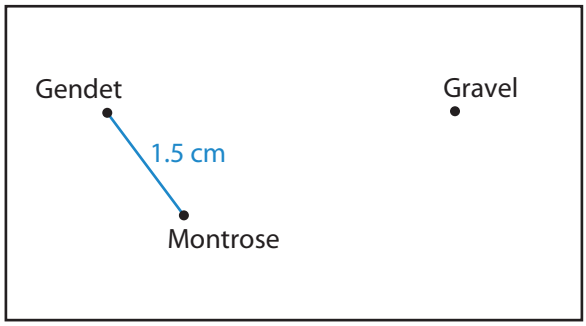
_____

8. Frances gets 6 paychecks in 12 weeks. How many paychecks does she get in 52 weeks?

_____

9. What is the actual distance between Gendet and Montrose? (Example 3)

_____



Scale: 1 centimeter = 16 kilometers

ESSENTIAL QUESTION CHECK-IN

10. How do you solve problems with proportions?

_____

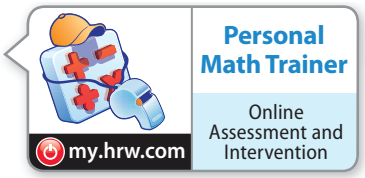
_____

_____

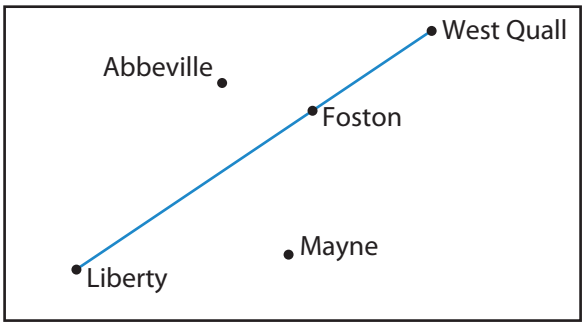
Name _____ Class _____ Date _____

7.2 Independent Practice

COMMON CORE 6.RP.3, 6.RP.3b



11. The scale of the map is missing. The actual distance from Liberty to West Quall is 72 miles, and it is 6 inches on the map.



a. What is the scale of the map?

_____

b. Foston is between Liberty and West Quall and is 4 inches from Liberty on the map. How far is Foston from West Quall?

_____

12. A punch recipe says to mix 4 cups pineapple juice, 8 cups orange juice, and 12 cups seltzer in order to make 18 servings of punch.

a. How many cups of each ingredient do you need to make 108 cups of punch?

_____ cups pineapple juice

_____ cups orange juice

_____ cups seltzer

b. How many servings can be made from 108 cups of punch? _____

c. For every cup of seltzer you use, how much orange juice do you use?

_____

13. On an airplane, there are two seats on the left side in each row and three seats on the right side. There are 90 seats on the right side of the plane.

a. How many seats are on the left side of the plane? _____

b. How many seats are there altogether? _____

14. Carrie and Krystal are taking a road trip from Greenville to North Valley. Each person has her own map, and the scales on the maps are different.

a. On Carrie's map, Greenville and North Valley are 4.5 inches apart. The scale on her map is 1 inch = 20 miles. How far is Greenville from North Valley?

_____

b. The scale on Krystal's map is 1 inch = 18 miles. How far apart are Greenville and North Valley on Krystal's map?

_____

15. **Multistep** A machine can produce 27 inches of ribbon every 3 minutes. How many feet of ribbon can the machine make in one hour? Explain.

_____

_____

_____

_____

_____

Marta, Loribeth, and Ira all have bicycles. The table shows the number of miles of each rider's last bike ride, as well as the time it took each rider to complete the ride.

	Distance of Last Ride (in miles)	Time Spent on Last Bike Ride (in minutes)
Marta	8	80
Loribeth	6	42
Ira	15	75

16. What is Marta's unit rate, in minutes per mile? _____

17. Whose speed was the fastest on their last bike ride? _____

18. If all three riders travel for 3.5 hours at the same speed as their last ride, how many total miles will the 3 riders travel altogether? Explain.

_____

_____

_____

_____

19. **Critique Reasoning** Jason watched a caterpillar move 10 feet in 2 minutes. Jason says that the caterpillar's unit rate is 0.2 feet per minute. Is Jason correct? Explain.

_____

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

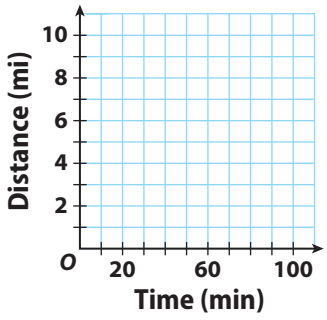
20. **Analyze Relationships** If the number in the numerator of a unit rate is 1, what does this indicate about the equivalent unit rates? Give an example.

_____

_____

21. **Multiple Representations** A boat travels at a constant speed. After 20 minutes, the boat has traveled 2.5 miles. The boat travels a total of 10 miles to a bridge.

a. Graph the relationship between the distance the boat travels and the time it takes.



b. How long does it take the boat to reach the bridge? Explain how you found it.

_____

_____

_____

_____

LESSON 7.3

Converting Within Measurement Systems

COMMON CORE 6.RP.3d

Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. *Also 6.RP.3*

ESSENTIAL QUESTION

How do you convert units within a measurement system?

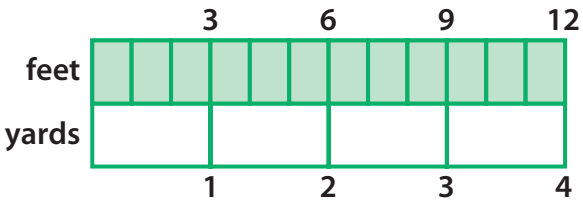
EXPLORE ACTIVITY Real World

COMMON CORE 6.RP.3d

Using a Model to Convert Units

The two most common systems of measurement are the customary system and the metric system. You can use a model to convert from one unit to another within the same measurement system.

STEP 1 Use the model to complete each statement below.



1 yard = 3 feet

2 yards = _____ feet

3 yards = _____ feet

4 yards = _____ feet

STEP 2 Rewrite your answers as ratios.

$\frac{\square \text{ feet}}{2 \text{ yards}} = \frac{3 \text{ feet}}{1 \text{ yard}}$ $\frac{\square \text{ feet}}{3 \text{ yards}} = \frac{3 \text{ feet}}{1 \text{ yard}}$ $\frac{\square \text{ feet}}{4 \text{ yards}} = \frac{3 \text{ feet}}{1 \text{ yard}}$

Since 1 yard = 3 feet, the ratio of feet to yards in any measurement is always $\frac{3}{1}$. This means any ratio forming a proportion with $\frac{3}{1}$ can represent a ratio of feet to yards.

$\frac{3}{1} = \frac{12}{4}$, so 12 feet = _____ yards. $\frac{3}{1} = \frac{54}{18}$, so _____ feet = 18 yards.

Reflect

1. **Communicate Mathematical Ideas** How could you draw a model to show the relationship between feet and inches?

_____

_____

_____

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Converting Units Using Proportions and Unit Rates

You can use ratios and proportions to convert both customary and metric units. Use the table below to convert from one unit to another within the same measurement system.

Customary Measurements		
Length	**Weight**	**Capacity**
1 ft = 12 in. 1 yd = 36 in. 1 yd = 3 ft 1 mi = 5,280 ft 1 mi = 1,760 yd	1 lb = 16 oz 1 T = 2,000 lb	1 c = 8 fl oz 1 pt = 2 c 1 qt = 2 pt 1 qt = 4 c 1 gal = 4 qt
Metric Measurements		
Length	**Mass**	**Capacity**
1 km = 1,000 m 1 m = 100 cm 1 cm = 10 mm	1 kg = 1,000 g 1 g = 1,000 mg	1 L = 1,000 mL

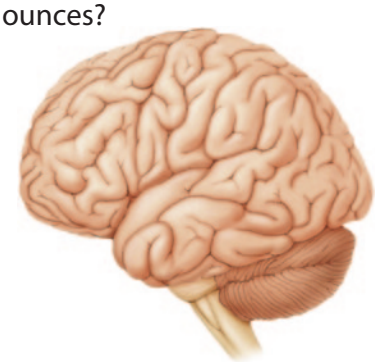
EXAMPLE 1

Real World

COMMON CORE 6.RP.3d

My Notes

A What is the weight of a 3-pound human brain in ounces?



Use a proportion to convert 3 pounds to ounces.

Use $\frac{16 \text{ ounces}}{1 \text{ pound}}$ to convert pounds to ounces.

STEP 1 Write a proportion.

$$\frac{16 \text{ ounces}}{1 \text{ pound}} = \frac{\square \text{ ounces}}{3 \text{ pounds}}$$

STEP 2 Use common denominators to write equivalent ratios.

$$\frac{16 \times 3}{1 \times 3} = \frac{\square}{3}$$ 3 is a common denominator.

$$\frac{48}{3} = \frac{\square}{3}$$ Equivalent rates with the same denominators have the same numerators.

$\square = 48$ ounces

The weight is 48 ounces.

B A moderate amount of daily sodium consumption is 2,000 milligrams. What is this mass in grams?

Use a proportion to convert 2,000 milligrams to grams.

Use $\frac{1{,}000 \text{ mg}}{1 \text{ g}}$ to convert milligrams to grams.

STEP 1 Write a proportion.

$$\frac{1{,}000 \text{ mg}}{1 \text{ g}} = \frac{2{,}000 \text{ mg}}{\square \text{ g}}$$

STEP 2 Write equivalent ratios.

Think: You can multiply 1,000 × 2 = 2,000. So multiply the denominator by the same number.

$$\frac{1{,}000 \times 2}{1 \times 2} = \frac{2{,}000}{\square}$$

$$\frac{2{,}000}{2} = \frac{2{,}000}{\square}$$

Equivalent ratios with the same numerators have the same denominators.

$\square$ = 2 grams

The mass is 2 grams.

Math Talk
Mathematical Practices
How would you convert 3 liters to milliliters?

YOUR TURN

2. The height of a doorway is 2 yards. What is the height of the doorway in inches? _____

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Converting Units by Using Conversion Factors

Another way to convert measurements is by using a conversion factor. A **conversion factor** is a ratio comparing two equivalent measurements.

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EXAMPLE 2

Real World

COMMON CORE 6.RP.3d

Elena wants to buy 2 gallons of milk but can only find quart containers for sale. How many quarts does she need?

You are converting to quarts from gallons.

STEP 1 Find the conversion factor.

Write 4 quarts = 1 gallon as a ratio: $\frac{4 \text{ quarts}}{1 \text{ gallon}}$

STEP 2 Multiply the given measurement by the conversion factor.

$$2 \text{ gallons} \cdot \frac{4 \text{ quarts}}{1 \text{ gallon}} = \square \text{ quarts}$$

$$2 \text{ gallons} \cdot \frac{4 \text{ quarts}}{1 \text{ gallon}} = 8 \text{ quarts}$$

Cancel the common unit.

Elena needs 8 quarts of milk.

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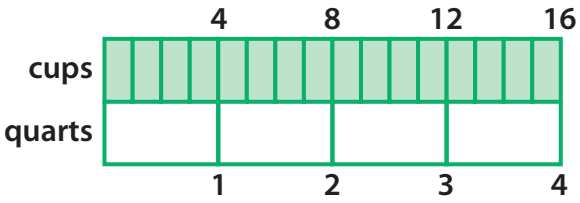
YOUR TURN

3. An oak tree is planted when it is 250 centimeters tall. What is this height in meters? _____

Guided Practice

Use the model below to complete each statement. (Explore Activity 1)



1. $\frac{4}{1} = \frac{12}{3}$, so 12 cups = _____ quarts

2. $\frac{4}{1} = \frac{48}{12}$, so _____ cups = 12 quarts

Use ratios and proportions to solve. (Example 1)

3. Mary Catherine makes 2 gallons of punch for her party. How many cups of punch did she make?

_____

4. An African elephant weighs 6 tons. What is the weight of the elephant in pounds?

_____

5. The distance from Jason's house to school is 0.5 kilometer. What is this distance in meters?

_____

6. The mass of a moon rock is 3.5 kilograms. What is the mass of the moon rock in grams?

_____

Use a conversion factor to solve. (Example 2)

7. 1.75 grams $\cdot \frac{1{,}000 \text{ mg}}{1 \text{ g}} =$ _____

8. 27 millimeters $\cdot \frac{1 \text{ cm}}{10 \text{ mm}} =$ _____

9. A package weighs 96 ounces. What is the weight of the package in pounds?

_____

10. A jet flies at an altitude of 52,800 feet. What is the height of the jet in miles?

_____

ESSENTIAL QUESTION CHECK-IN

11. How do you convert units within a measurement system?

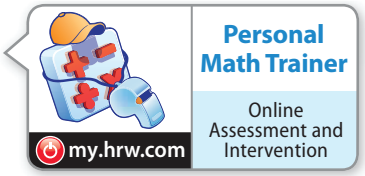
_____

_____

Name _____ Class _____ Date _____

7.3 Independent Practice

COMMON CORE 6.RP.3d



12. What is a conversion factor that you can use to convert gallons to pints? How did you find it?

_____

13. Three friends each have some ribbon. Carol has 42 inches of ribbon, Tino has 2.5 feet of ribbon, and Baxter has 1.5 yards of ribbon. Express the total length of ribbon the three friends have in inches, feet and yards.

_____ inches = _____ feet = _____ yards

14. Suzanna wants to measure a board, but she doesn't have a ruler to measure with. However, she does have several copies of a book that she knows is 17 centimeters tall.

a. Suzanna lays the books end to end and finds that the board is the same length as 21 books. How many centimeters long is the board?

_____

b. Suzanna needs a board that is at least 3.5 meters long. Is the board long enough? Explain.

_____

_____

Sheldon needs to buy 8 gallons of ice cream for a family reunion. The table shows the prices for different sizes of two brands of ice cream.

	Price of small size	Price of large size
Cold Farms	$2.50 for 1 pint	$4.50 for 1 quart
Cone Dreams	$4.25 for 1 quart	$9.50 for 1 gallon

15. Which size container of Cold Farm ice cream is the better deal for Sheldon? Explain.

_____

_____

_____

16. **Multistep** Which size and brand of ice cream is the best deal?

_____

17. In Beijing in 2008, the Women's 3,000 meter Steeplechase became an Olympic event. What is this distance in kilometers? _____

18. How would you convert 5 feet 6 inches to inches? _____

_____

H.O.T. FOCUS ON HIGHER ORDER THINKING

19. **Analyze Relationships** A Class 4 truck weighs between 14,000 and 16,000 pounds.

a. What is the weight range in tons? _____

b. If the weight of a Class 4 truck is increased by 2 tons, will it still be classified as a Class 4 truck? Explain.

_____

_____

20. **Persevere in Problem Solving** A football field is shown at right.

a. What are the dimensions of a football field in feet?

_____

$53\frac{1}{3}$ yd

120 yd

b. A chalk line is placed around the perimeter of the football field. What is the length of this line in feet?

_____

c. About how many laps around the perimeter of the field would equal 1 mile? Explain.

_____

_____

21. **Look for a Pattern** What is the result if you multiply a number of cups by $\frac{8 \text{ fl oz}}{1 \text{ cup}}$ and then multiply the result by $\frac{1 \text{ cup}}{8 \text{ fl oz}}$? Give an example.

_____

_____

22. **Make a Conjecture** 1 hour = 3,600 seconds and 1 mile = 5,280 feet. Make a conjecture about how you could convert a speed of 15 miles per hour to feet per second. Then convert.

_____

_____

_____

Work Area

LESSON 7.4

Converting Between Measurement Systems

COMMON CORE 6.RP.3d

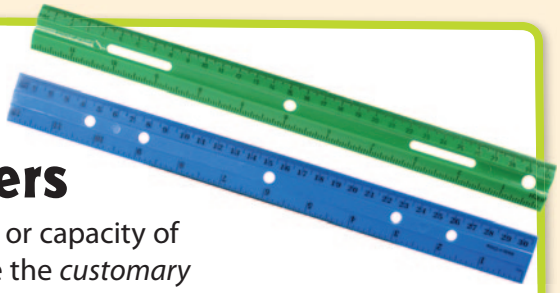
Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. *Also 6.RP.3, 6.RP.3b*

ESSENTIAL QUESTION

How can you use ratios and proportions to convert measurements?

EXPLORE ACTIVITY Real World

COMMON CORE 6.RP.3d



Converting Inches to Centimeters

Measurements are used when determining the length, weight, or capacity of an object. The two most common systems of measurement are the *customary system* and the *metric system*.

The table shows equivalencies between the customary and metric systems. You can use these equivalencies to convert a measurement in one system to a measurement in the other system.

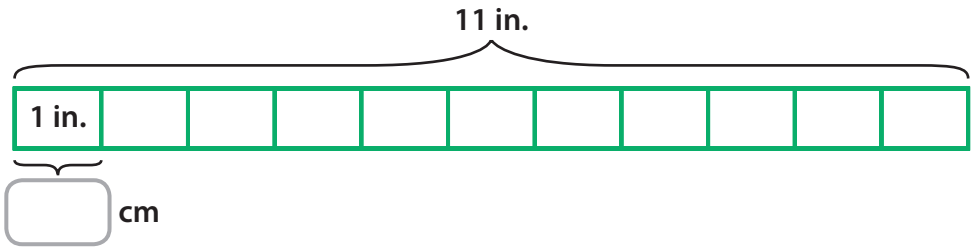
Length	Weight/Mass	Capacity
1 inch = 2.54 centimeters 1 foot ≈ 0.305 meter 1 yard ≈ 0.914 meter 1 mile ≈ 1.61 kilometers	1 ounce ≈ 28.4 grams 1 pound ≈ 0.454 kilogram	1 fluid ounce ≈ 29.6 milliliters 1 quart ≈ 0.946 liter 1 gallon ≈ 3.79 liters

Most conversions are approximate, as indicated by the symbol ≈.

The length of a sheet of paper is 11 inches. What is this length in centimeters?

A You can use a bar diagram to solve this problem. Each part represents 1 inch.

1 inch = _____ centimeter(s)



B How does the diagram help you solve the problem?

_____

C 11 inches = _____ centimeters

EXPLORE ACTIVITY *(cont'd)*

Reflect

1. **Communicate Mathematical Ideas** Suppose you wanted to use a diagram to convert ounces to grams. Which unit would the parts in your diagram represent?

_____

_____

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Using Conversion Factors

Another way to convert measurements is by using a ratio called a *conversion factor*. A **conversion factor** is a ratio of two equivalent measurements. Since the two measurements in a conversion factor are equivalent, a conversion factor is equal to 1.

EXAMPLE 1

Real World

COMMON CORE 6.RP.3d

While lifting weights, John adds 11.35 kilograms to his bar. About how many pounds does he add to his bar?

STEP 1 Find the conversion factor.

1 pound $\approx$ 0.454 kilogram

Write the conversion factor as a ratio: $\frac{1 \text{ pound}}{0.454 \text{ kilogram}}$

STEP 2 Convert the given measurement.

kilograms	$\times$	conversion factor	$=$	pounds

$$11.35 \text{ kilograms} \times \frac{1 \text{ pound}}{0.454 \text{ kilogram}} \approx 25 \text{ pounds}$$

John adds about 25 pounds to his bar.

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YOUR TURN

2. 6 quarts $\approx$ _____ liters
3. 14 feet $\approx$ _____ meters
4. 255.6 grams $\approx$ _____ ounces
5. 7 liters $\approx$ _____ quarts

Using Proportions to Convert Measurements

You can also convert a measurement from one unit to another by using a proportion. First write the conversion factor as a ratio, then multiply by a form of 1 to generate an equivalent ratio. Recall that two equal ratios form a proportion.

Proportions: $\frac{3 \text{ inches}}{2 \text{ feet}} = \frac{6 \text{ inches}}{4 \text{ feet}}$ $\frac{5}{10} = \frac{1}{2}$

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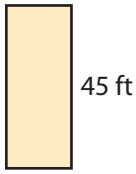
EXAMPLE 2

Real World

COMMON CORE 6.RP.3, 6.RP.3b, 6.RP.3d

Bob's driveway is 45 feet long by 18 feet wide. He plans to pave the entire driveway. The asphalt paving costs $24 per square meter. What will be the total cost of the paving?



STEP 1 First find the dimensions of the driveway in meters.

Convert each measurement to meters.
Use 1 foot ≈ 0.305 meter.

$$\frac{1 \text{ foot}}{0.305 \text{ meter}} \overset{\times 45}{=} \frac{45 \text{ feet}}{13.725 \text{ meters}}$$

$$\frac{1 \text{ foot}}{0.305 \text{ meter}} \overset{\times 18}{=} \frac{18 \text{ feet}}{5.49 \text{ meters}}$$

Length ≈ 13.725 meters

Width ≈ 5.49 meters

The length and width are approximate because the conversion between feet and meters is approximate.

STEP 2 Find the area in square meters.

Area = length × width
= 13.725 × 5.49
= 75.35 square meters

STEP 3 Now find the total cost of the paving.

square meters × cost per square meter = total cost
75.35 × $24 = $1,808.40

Math Talk
Mathematical Practices
How much does the paving cost per square foot? Explain.

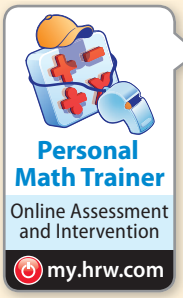
My Notes

Reflect

6. **Error Analysis** Yolanda found the area of Bob's driveway in square meters as shown. Explain why Yolanda's answer is incorrect.

Area = 45 × 18 = 810 square feet

810 square feet × $\frac{0.305 \text{ meter}}{1 \text{ foot}}$ ≈ 247.1 square meters



YOUR TURN

7. A flower bed is 2 meters wide and 3 meters long. What is the area of the flower bed in square feet? Round your converted dimensions and your final answer to the nearest hundredth.

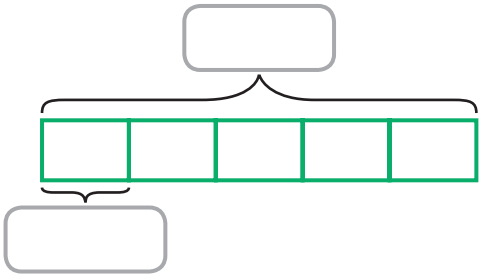
_____ square feet

Guided Practice

Complete each diagram to solve the problem. (Explore Activity)

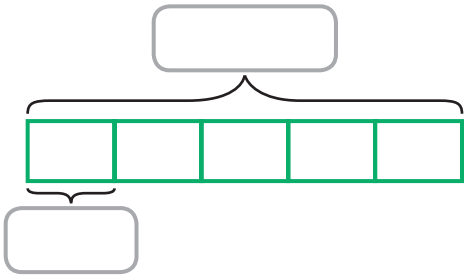
1. Kate ran 5 miles. How far did she run in kilometers?

5 miles = _____ kilometers



2. Alex filled a 5-gallon jug with water. How many liters of water are in the container?

5 gallons ≈ _____ liters



Use a conversion factor to convert each measurement. (Example 1 and 2)

3. A ruler is 12 inches long. What is the length of this ruler in centimeters?

_____ centimeters

4. A kitten weighs 4 pounds. What is the approximate mass of the kitten in kilograms?

_____ kilograms

Use a proportion to convert each measurement. (Example 2)

5. 20 yards ≈ _____ meters

6. 12 ounces ≈ _____ grams

7. 5 quarts ≈ _____ liters

8. 400 meters ≈ _____ yards

9. 10 liters ≈ _____ gallons

10. 137.25 meters ≈ _____ feet

11. 165 centimeters ≈ _____ inches

12. 10,000 kilometers ≈ _____ miles

ESSENTIAL QUESTION CHECK-IN

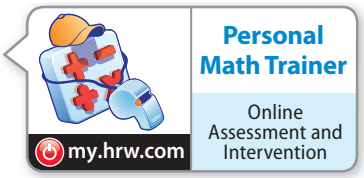
13. Write a proportion that you can use to convert 60 inches to centimeters.

_____

Name _____ Class _____ Date _____

7.4 Independent Practice

COMMON CORE 6.RP.3, 6.RP.3b, 6.RP.3d



Tell which measure is greater.

14. Six feet or two meters _____

15. One inch or one centimeter _____

16. One yard or one meter _____

17. One mile or one kilometer _____

18. One ounce or one gram _____

19. One quart or one liter _____

20. 10 pounds or 10 kilograms _____

21. Four liters or one gallon _____

22. Two miles or three kilometers _____

23. What is the limit in kilograms?

_____

24. What is the speed limit in miles per hour?

_____

25. Which container holds more, a half-gallon milk jug or a 2-liter juice bottle?

_____

26. The label on a can of lemonade gives the volume as 12 fl oz, or 355 mL. Verify that these two measurements are nearly equivalent.

_____

27. The mass of a textbook is about 1.25 kilograms. About how many pounds is this?

_____

28. **Critique Reasoning** Michael estimated his mass as 8 kilograms. Is his estimate reasonable? Justify your answer.

_____

29. Your mother bought a three-liter bottle of water. When she got home, she discovered a small leak in the bottom and asked you to find a container to transfer the water into. All you could find were two half-gallon jugs.

a. Will your containers hold all of the water?

_____

_____

_____

b. **What If?** Suppose an entire liter of water leaked out in the car. In that case, would you be able to fit all of the remaining water into one of the half-gallon jugs?

_____

_____

_____

_____

30. The track team ran a mile and a quarter during their practice.

How many kilometers did the team run? _____

31. A countertop is 16 feet long and 3 feet wide.

a. What is the area of the countertop in square meters? _____

b. Tile costs $28 per square meter. How much will it cost to cover the countertop with new tile? $ _____

32. At a school picnic, your teacher asks you to mark a field every ten yards so students can play football. The teacher accidentally gave you a meter stick instead of a yard stick. How far apart in meters should you mark the lines if you still want them to be in the right places?

_____

33. You weigh a gallon of 2% milk in science class and learn that it is approximately 8.4 pounds. You pass the milk to the next group, and then realize that your teacher wanted an answer in kilograms, not pounds. Explain how you can adjust your answer without weighing the milk again. Then give the weight in kilograms.

_____

H.O.T. FOCUS ON HIGHER ORDER THINKING

34. **Analyze Relationships** Annalisa, Keiko, and Stefan want to compare their heights. Annalisa is 64 inches tall. Stefan tells her, "I'm about 7.5 centimeters taller than you." Keiko knows she is 1.5 inches shorter than Stefan. Give the heights of all three people in both inches and centimeters to the nearest half unit.

_____

_____

_____

_____

35. **Communicate Mathematical Ideas** Mikhael wanted to rewrite the conversion factor "1 yard $\approx$ 0.914 meter" to create a conversion factor to convert meters to yards. He wrote "1 meter $\approx$ _____." Tell how Mikhael should finish his conversion, and explain how you know.

_____

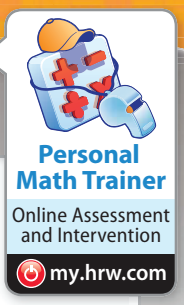
_____

_____

Work Area

MODULE QUIZ

Ready to Go On?



7.1 Ratios, Rates, Tables, and Graphs

1. Charlie runs laps around a track. The table shows how long it takes him to run different numbers of laps. How long would it take Charlie to run 5 laps?

Number of Laps	2	4	6	8	10
Time (min)	10	20	30	40	50

_____

7.2 Solving Proportionality Problems

2. Emily is entering a bicycle race for charity. Her mother pledges $0.40 for every 0.25 mile she bikes. If Emily bikes 15 miles, how much will her mother donate? _____

3. Rob is saving to buy a new MP3 player. For every $15 he earns babysitting, he saves $6. On Saturday, Rob earned $75 babysitting. How much money did he save? _____

7.3 Within Measurement Systems

Convert each measurement.

4. 18 meters = _____ centimeters

5. 5 pounds = _____ ounces

6. 6 quarts = _____ fluid ounces

7. 9 liters = _____ milliliters

7.4 Converting Between Measurement Systems

Convert each measurement.

8. 5 inches = _____ centimeters

9. 198.9 grams ≈ _____ ounces

10. 8 gallons ≈ _____ liters

11. 12 feet ≈ _____ meters

ESSENTIAL QUESTION

12. Write a real-world problem that could be solved using a proportion.

_____

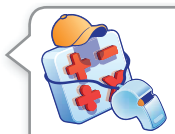
_____

_____

COMMON CORE

MODULE 7 MIXED REVIEW

Assessment Readiness



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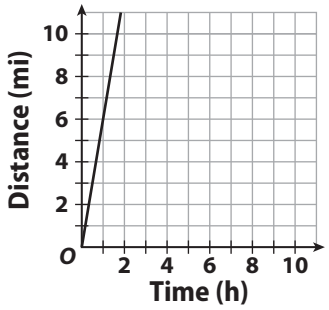
Selected Response

1. The graph below represents the distance Manuel walks over several hours.



Which is an ordered pair on the line?

Ⓐ (2.5, 14) Ⓒ (2.25, 12)

Ⓑ (1.25, 5) Ⓓ (1.5, 9)

2. Jonah's house and his grandparents' house are 8,046.72 meters apart. What is this distance in miles?

Ⓐ 4 miles Ⓒ 7 miles

Ⓑ 5 miles Ⓓ 8 miles

3. Megan is making bracelets to sell to earn money for the local animal shelter. It takes her $\frac{1}{4}$ hour to pick out all the beads and $\frac{1}{10}$ hour to string them. This week, she only has $5\frac{1}{4}$ hours to make bracelets. How many bracelets will Megan be able to make?

Ⓐ 10 bracelets Ⓒ 15 bracelets

Ⓑ 12 bracelets Ⓓ 21 bracelets

4. Rosa can run 4 miles in 56 minutes. How many miles does Rosa run if she runs for 42 minutes?

Ⓐ 2 miles Ⓒ 3.5 miles

Ⓑ 3 miles Ⓓ 5 miles

5. The table below shows the number of petals and leaves for different numbers of flowers.

Petals	5	10	15	20
Leaves	2	4	6	8

How many petals are present when there are 12 leaves?

Ⓐ 25 petals Ⓒ 35 petals

Ⓑ 30 petals Ⓓ 36 petals

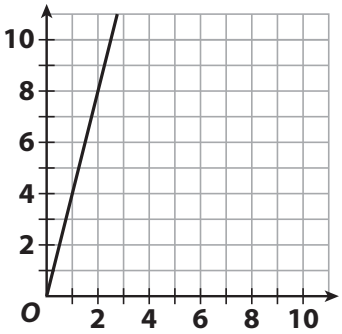
6. A recipe calls for 3 cups of sugar and 9 cups of water. How many cups of water should be used with 2 cups of sugar?

Ⓐ 3 cups Ⓒ 6 cups

Ⓑ 4 cups Ⓓ 8 cups

Mini-Task

7. The unlabeled graph shows the relationship between two customary units of measure. Only two pairs of units can be represented by the graph.



a. Determine the possible pairs of units.

_____

_____

b. Describe the relationship for each pair.

_____

_____