Distance and Area in the Coordinate Plane

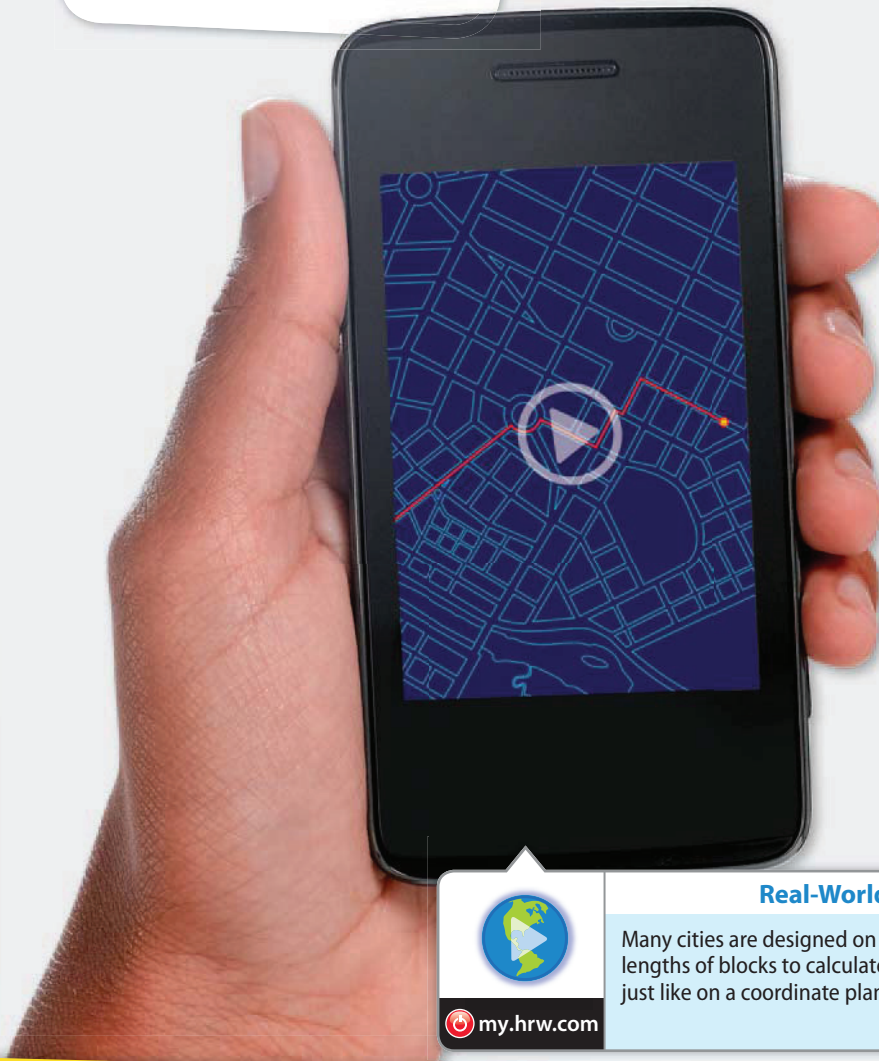**LESSON 14.1**
Distance in the Coordinate Plane
COMMON CORE   6.NS.6b, 6.NS.8

LESSON 14.2
Polygons in the Coordinate Plane
COMMON CORE   6.G.3

? ESSENTIAL QUESTION

What steps might you take to solve a polygon problem given the coordinates of its vertices?

Real-World Video

Many cities are designed on a grid. You can use the lengths of blocks to calculate perimeters and areas just like on a coordinate plane.

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Are YOU Ready?

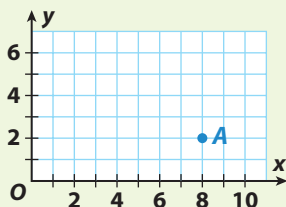Complete these exercises to review skills you will need for this module.

Graph Ordered Pairs
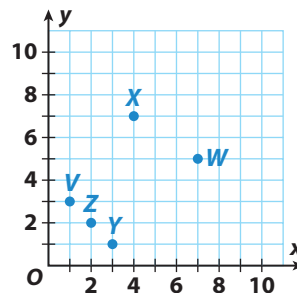
EXAMPLE   Find the coordinates for Point *A*.

Start at O.
Count 8 units to the right and 2 units up from O.

The coordinates for Point *A* are (8, 2).

Write the ordered pair for each point shown on the graph.

1. Point *V* _____

2. Point *W* _____

3. Point *X* _____

4. Point *Y* _____

5. Point *Z* _____

Identify Polygons

EXAMPLE   Name the type of polygon.

Count the number of sides.
Compare the sides.
Compare the angles.

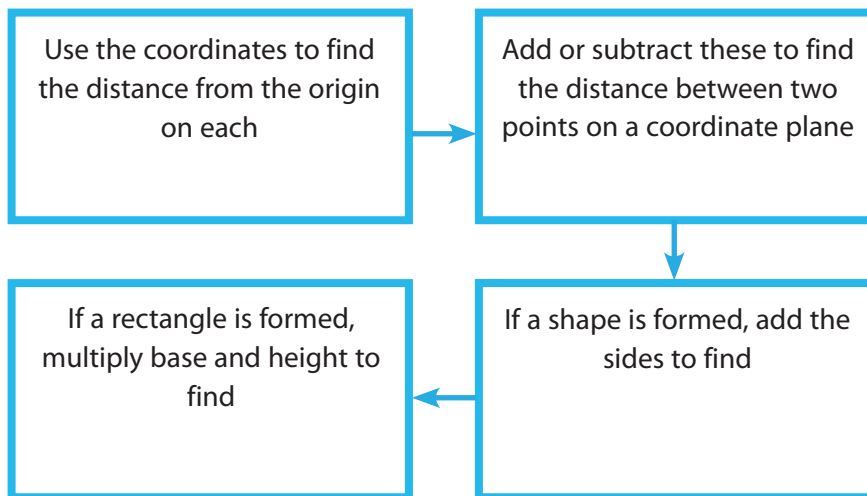There are 4 congruent sides and angles. The shape is a rhombus.

Name each figure. Choose from hexagon, isosceles triangle, right triangle, and trapezoid.

6. _____

7. _____

Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the graphic.

|	
Use the coordinates to find the distance from the origin on each	Add or subtract these to find the distance between two points on a coordinate plane
If a rectangle is formed, multiply base and height to find	If a shape is formed, add the sides to find

Understand Vocabulary

Complete the sentences using the preview words.

1. A corner of a rectangle is called a _____.

2. How an image would appear in a mirror is called a _____.

3. A two-dimensional shape with straight sides is

   a _____.

Vocabulary

Review Words
- ✔ absolute value *(valor absoluto)*
- ✔ area *(área)*
- ✔ axis *(eje)*
  - coordinate plane *(plano cartesiano)*
- ✔ perimeter *(perímetro)*

Preview Words
- polygon *(polígono)*
- reflection *(reflexión)*
- vertex, vertices *(vértice, vértices)*

Active Reading

Two-Panel Flip Chart Create a two-panel flip chart to help you understand the concepts in Lesson 14.1. Label one flap "Reflection across the *x*-axis." Label the other flap "Reflection across the *y*-axis." Write important ideas about each type of reflection under the appropriate flap.
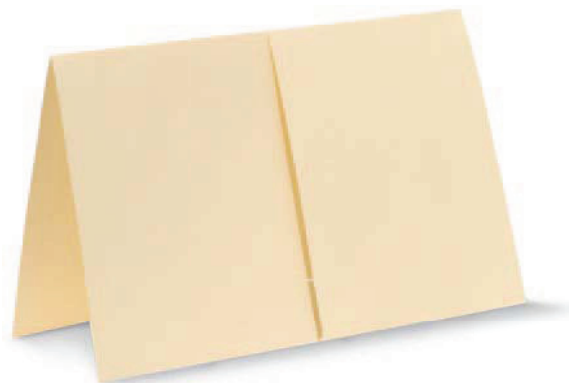
Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

COMMON CORE **6.NS.8**

Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

What It Means to You

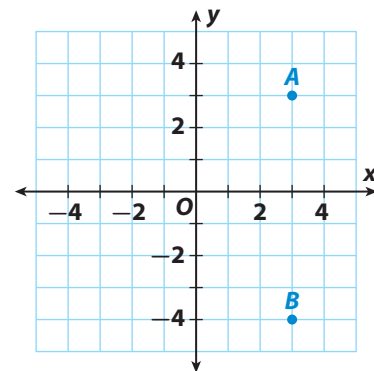You will find the distance between two points on a coordinate plane.

UNPACKING EXAMPLE 6.NS.8

Find the distance between points A and B.

Add the absolute values of the y-coordinates.

$$= |3| + |-4|$$
$$= 3 + 4$$
$$= 7$$

The distance between points A and B is 7 units.

COMMON CORE **6.G.3**

Draw polygons in the coordinate plane given coordinates for the vertices; …Apply these techniques in the context of solving real-world and mathematical problems.

What It Means to You

You can draw polygons on a coordinate plane by plotting the vertices and connecting them.

UNPACKING EXAMPLE 6.G.3

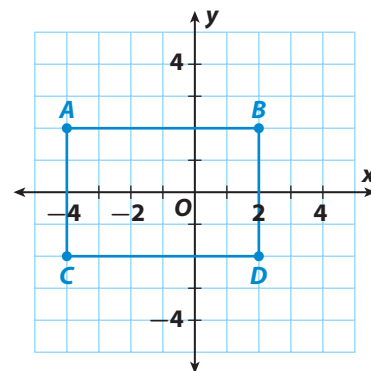What type of polygon can you make by plotting these points?

Point A $(-4, 2)$
Point B $(2, 2)$
Point C $(-4, -2)$
Point D $(2, -2)$
Connect each point.
The points make a rectangle.

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Distance in the Coordinate Plane

? ESSENTIAL QUESTION

How can you use absolute value to find the distance between two points with the same *x*- or *y*-coordinates?

EXPLORE ACTIVITY 1 COMMON CORE 6.NS.6b

Reflecting in the Coordinate Plane

A point on a coordinate plane can be *reflected* across an axis. The **reflection** is located on the opposite side of the axis, at the same distance from the axis.

> Hold your paper up to the light if necessary to see the reflection.

Draw a coordinate plane on graph paper. Label both axes from −10 to 10.

A Graph (3, −2). Then fold your coordinate plane along the *y*-axis and find the reflection of (3, −2). Record the coordinates of the new point in the table.

B Unfold your coordinate plane. Then fold it along the *x*-axis and find the reflection of (3, −2). Record the coordinates of the new point in the table.

C Choose three additional points and repeat **A** and **B** .

Point	Reflected across *y*-axis	Reflected across *x*-axis
(3, −2)		

Reflect

1. What is the relationship between the coordinates of a point and the coordinates of its reflection across each axis?

_____

_____

_____

2. Conjecture A point is reflected across the *y*-axis. Then the reflected point is reflected across the *x*-axis. How will the coordinates of the final point be related to the coordinates of the original point?

_____

_____

Finding Distances in the Coordinate Plane

You can also use absolute values to find distances between two points that have the same *x*-coordinates or the same *y*-coordinates on a coordinate plane.

EXAMPLE 1

COMMON CORE 6.NS.6b

Find each distance.

A What is the distance between point *A* and point *B*?

STEP 1 Find the distance between point *A* and the *x*-axis.

The *y*-coordinate is 3, so point *A* is $|3|$ units from the *x*-axis.

STEP 2 Find the distance between point *B* and the *x*-axis.

The *y*-coordinate of *B* is -2, so point *B* is $|-2| = 2$ units from the *x*-axis.

STEP 3 Find the sum of the distances.

Distance from *A* to $B = |3| + |-2| = 3 + 2 = 5$ units.

B What is the distance between point *D* and point *C*?

STEP 1 Find the distance between point *D* and the *y*-axis.
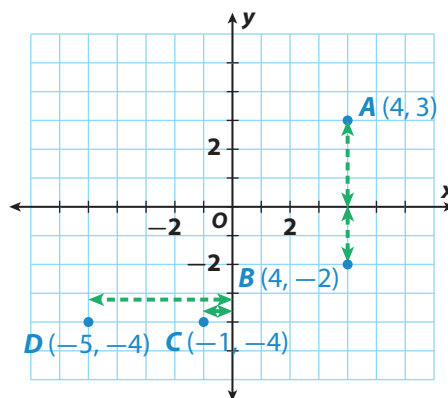
Point *D* is $|-5| = 5$ units from the *y*-axis.

STEP 2 Find the distance between point *C* and the *y*-axis.

Point *C* is $|-1| = 1$ unit from the *y*-axis.

STEP 3 Find the distance between *C* and *D* by finding this difference:

Distance of *D* from the *y*-axis $-$ distance of *C* from the *y*-axis

$|-5| - |-1| = 4$ units

Math Talk
Mathematical Practices

How is the distance between a point and its reflection across an axis related to the distance between the point and the axis? Explain.

YOUR TURN

Find the distance between each pair of points.

3. $E(-4, 7)$ and $F(5, 7)$ _____

4. $G(0, -5)$ and $H(0, -10)$ _____

Solving Distance Problems

You can solve problems using the distance between points on a grid.
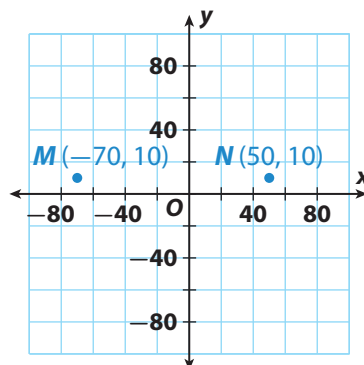
EXAMPLE 2 — Problem Solving

COMMON CORE 6.NS.8

The coordinate plane represents a map. Each grid unit represents 20 miles. A retail company has warehouses at $M(-70, 10)$ and $N(50, 10)$. How long does it take a truck that drives 40 miles per hour to travel from warehouse M to warehouse N?

Analyze Information

Identify the important information.

- One warehouse is located at $M(-70, 10)$. The other is at $N(50, 10)$.
- A truck drives from M to N at a speed of 40 miles per hour.

Formulate a Plan

- Find the distance between M and N by adding the absolute values of the x-coordinates of the points.
- Find the time it takes the truck to drive this distance by using the relationship, distance = rate · time.

Solve

Add the absolute values of the x-coordinates to find the distance between point M and point N on the grid.

$$|-70| + |50| = 70 + 50 = 120$$

The warehouses are 120 miles apart.

The truck drives 120 miles at 40 mi/h. Because $120 = 40(3)$, it takes the truck 3 hours to travel from M to N.

Justify and Evaluate

You found the sum of the absolute values of the x-coordinates to find the horizontal distance on the grid. Then you used distance = rate · time to find the time it takes to drive that distance.

YOUR TURN

5. A store is located at $P(50, -30)$. How long will it take a truck driving at 50 miles per hour to drive from warehouse N to this store?

1. The point (5, −2) is reflected across the *x*-axis. What are the coordinates of the reflection? (Explore Activity)

_____

2. The point (−6, 8) is reflected across the *y*-axis. What are the coordinates of the reflection? (Explore Activity)
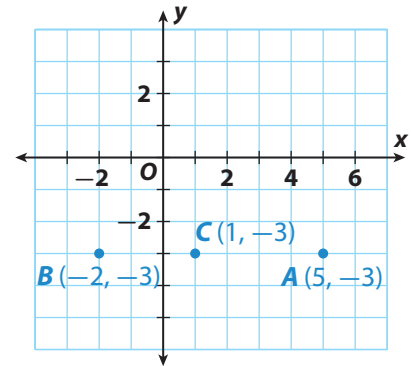
_____

Use the coordinate plane. (Example 1)

3. The distance between point *A* and point *B* is

$$\left|\boxed{}\right| + \left|\boxed{}\right| = \boxed{} + \boxed{} = \boxed{} \text{ units.}$$

4. The distance between point *A* and point *C* is

$$\left|\boxed{}\right| - \left|\boxed{}\right| = \boxed{} - \boxed{} = \boxed{} \text{ units.}$$

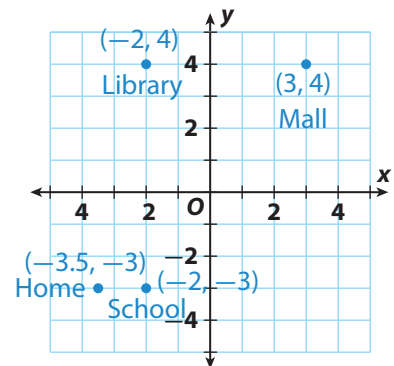*C* (1, −3)   *B* (−2, −3)   *A* (5, −3)

5. Plot the reflection of point *C* across the *y*-axis. What is the distance between point *C* and its reflection? _____

6. Plot the reflection of point *A* across the *x*-axis. What is the distance of the reflection from the *x*-axis? _____

Use the map shown. Each grid on the map represents 1 city block. (Example 2)

(−2, 4) Library   (3, 4) Mall   (−3.5, −3) Home   (−2, −3) School

7. Yoko walks from the library to the mall. How many city blocks does she walk? _____

8. If Yoko walks 1 block in 3 minutes, how long does it take her to walk from the school to the library? How long does it take her to walk from home to school?

_____

? ESSENTIAL QUESTION CHECK-IN

9. How do you use absolute value to find the distance between two points that have the same *x*-coordinates but different *y*-coordinates?

_____

_____

_____

_____

14.1 Independent Practice

COMMON CORE 6.NS.6b, 6.NS.8

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Use the coordinate plane.

10. Plot the reflection of point *A* across the *x*-axis. What are the coordinates of the reflection of point *A* across the *x*-axis? What is the distance between point *A* and its reflection?

_____

11. How can you plot the reflection of point *A* across the *y*-axis? Give the coordinates of the reflection across the *y*-axis, and tell how many units the reflection is from point *A*.

_____

_____

_____

A(1.5, −2.5)
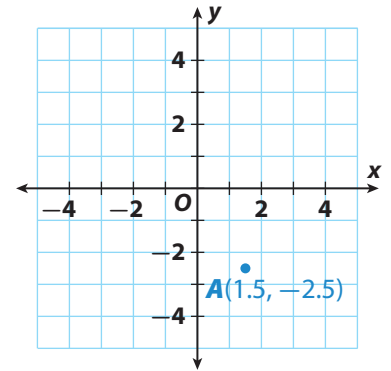
Find the coordinates of each point after the described reflection. Give the distance between each point and its reflection.

12. *R*(−5, 8) is reflected across the *x*-axis. _____

13. *S*(−7, −3) is reflected across the *y*-axis. _____

14. *T*(8, 2) is reflected across the *x*-axis. _____

15. *U*(2.4, −1) is reflected across the *y*-axis _____

Pedro uses a coordinate system to map the locations of some tourist locations in a large city. Each grid unit represents one mile.

Historic Village $\left(-4\frac{1}{2}, 5\frac{1}{4}\right)$ Water Park $\left(0, 5\frac{1}{4}\right)$

Science Center $\left(-4\frac{1}{2}, -1\frac{3}{4}\right)$ Museum $\left(2, -1\frac{3}{4}\right)$

16. The planetarium, which is not marked on the map, is halfway between the historic village and the science center. What are its coordinates?

_____

17. Pedro wants to walk from the historic village to the science center. Then he will walk from the science center to the museum. If he walks at a speed of $4\frac{1}{2}$ miles per hour, how long will it take him?

_____
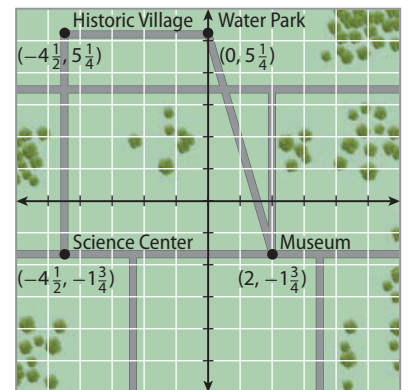
18. Pedro is staying at a hotel whose location is a reflection across the *x*-axis of the museum's location. What are the coordinates of the location of Pedro's hotel?

_____

19. Communicate Mathematical Ideas Deirdre plotted a point *D* in Quadrant IV. After she reflected the point across an axis, the reflection was in Quadrant III. Give possible coordinates for point *D* and its reflection, and tell why you chose these coordinates.

_____

_____

_____

_____

20. Explain the Error Jason plotted the points (4, 4) and (−4, −4) on a coordinate plane. He says that the distance between the two points is 8 units because $|4| + |-4| = 8$. What mistake is Jason making?

_____

_____

_____

21. Look for a Pattern A point is reflected over the *x*-axis and then reflected again over the *y*-axis. Will the coordinates after these two reflections be the same or different if the point is first reflected over the *y*-axis and then over the *x*-axis? Use an example to support your answer.

_____

_____

_____

_____

_____

22. Explain the Error Bentley states that the distance between $R(-8, -3.5)$ and $S(-8, -12)$ is $|-12| + |-3.5| = 15.5$ units. Is Bentley correct? Explain your answer. If Bentley is not correct, explain how to find the correct distance between the points.

_____

_____

_____

_____

Polygons in the Coordinate Plane

? ESSENTIAL QUESTION

How can you solve problems by drawing polygons in the coordinate plane?

EXPLORE ACTIVITY Real World

COMMON CORE 6.G.3

Polygons in the Coordinate Plane

A **polygon** is a closed plane figure formed by three or more line segments that meet only at their endpoints. A **vertex** is the point where two sides of a polygon meet. The *vertices* of a polygon can be represented as ordered pairs, and the polygon can then be drawn in the coordinate plane.

Sheila wants to make a pattern of two different tile shapes on a floor. She first graphs the shapes on a coordinate plane.

A Plot these points to form one of the tile shapes:

$A(3, 5)$, $B(4, 6)$, $C(5, 5)$, $D(4, 4)$

Connect the points in order.

The polygon formed is a(n) _____.

B Plot these points to form the other tile shape:

$P(-5, 2)$, $Q(-4, 3)$, $R(0, 3)$, $S(1, 2)$,

$T(1, -2)$, $U(0, -3)$, $V(-4, -3)$, $W(-5, -2)$

Connect the points in order.

The polygon formed is a(n) _____.
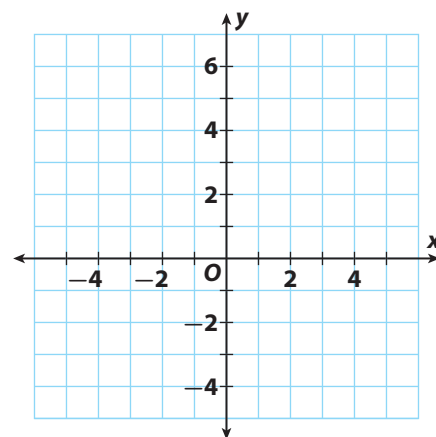
Reflect

1. How is the number of vertices related to the number of sides of the polygon and to the type of polygon? Give two examples.

_____

_____
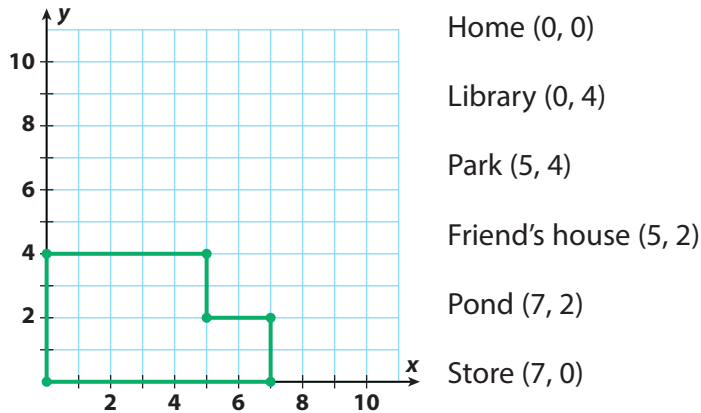
_____

_____

Finding Perimeter in the Coordinate Plane

You can use what you know about finding lengths in the coordinate plane to find the perimeter of a polygon.

EXAMPLE 1 Real World

The grid shows the path Tommy followed when he walked from his home at (0, 0) to various locations and back home again. If each grid square represents one block, how many blocks did he walk?

Home (0, 0)

Library (0, 4)

Park (5, 4)

Friend's house (5, 2)

Pond (7, 2)

Store (7, 0)

STEP 1 Find each distance. Each grid unit represents one block.

Tommy's home (0, 0) to the library (0, 4) is
$|4| - 0 = 4 - 0 = 4$ blocks.

The library (0, 4) to the park (5, 4) is $|5| - 0 = 5 - 0 = 5$ blocks.

The park (5, 4) to Tommy's friend's house (5, 2) is
$|4| - |2| = 4 - 2 = 2$ blocks.

Tommy's friend's house (5, 2) to the pond (7, 2) is
$|7| - |5| = 7 - 5 = 2$ blocks.

The pond (7, 2) to the store (7, 0) is $|2| - 0 = 2 - 0 = 2$ blocks.

The store (7, 0) to Tommy's home (0, 0) is
$|7| - 0 = 7 - 0 = 7$ blocks.

STEP 2 Find the sum of the distances.

Tommy walked $4 + 5 + 2 + 2 + 2 + 7 = 22$ blocks.

YOUR TURN

2. Suppose the next day Tommy walks from his home to the mall at (0, 8), and then walks to a movie theater at (7, 8). After leaving the theater Tommy walks to the store at (7, 0) before returning home.

   How far does he walk? _____ blocks
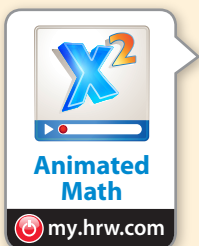
Finding Area in the Coordinate Plane

You can use familiar area formulas to find areas of polygons in the coordinate plane.

EXAMPLE 2 · Real World · COMMON CORE 6.G.3

Caleb is planning a new deck for his house. He graphs the deck as polygon *ABCDEF* on a coordinate plane in which each grid unit represents one foot. The vertices of the polygon are *A*(1, 0), *B*(3, 2), *C*(3, 5), *D*(8, 5), *E*(8, 2), and *F*(6, 0). What is the area of Caleb's deck?

STEP 1 Graph the vertices, and connect them in order.

Draw a horizontal dashed line segment to divide the polygon into two quadrilaterals—a rectangle and a parallelogram.
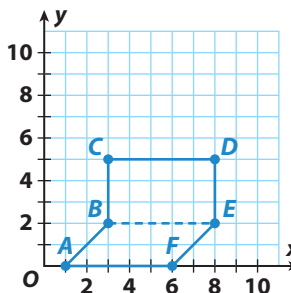
STEP 2 Find the area of the rectangle using the length of segment *BE* as the base *b* and the length of segment *BC* as the height *h*.

$b = |8| - |3| = 5$ feet $h = |5| - |2| = 3$ feet
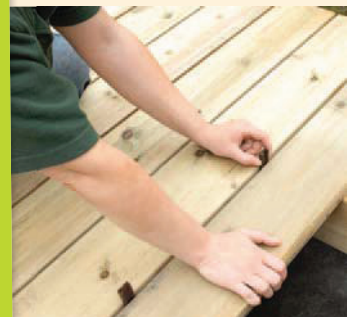
$A = bh = 5 \cdot 3 = 15$ square feet

STEP 3 Find the area of the parallelogram using the length of segment *AF* as the base. Use the length of a segment from *F*(6, 0) to the point (6, 2) as the height *h*.

$b = |6| - |1| = 5$ feet $h = |2| - 0 = 2$ feet

$A = bh = 5 \cdot 2 = 10$ square feet

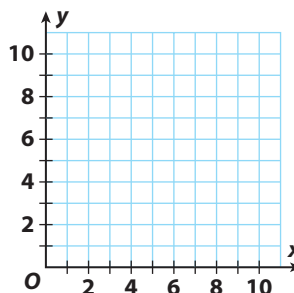**STEP 4** Add the areas to find the total area of the deck.

$A = 15 + 10 = 25$ square feet

YOUR TURN

3. The vertices of a polygon are *L*(1, 2), *M*(1, 6), *N*(7, 6), *O*(7, 2), *P*(5, 0), and *Q*(3, 0). Graph the polygon. Then find its area.

$A = $ _____ square units

A gardener uses a coordinate grid to design a new garden. The gardener uses polygon *WXYZ* on the grid to represent the garden. The vertices of this polygon are *W*(3, 3), *X*(−3, 3), *Y*(−3, −3), and *Z*(3, −3). Each grid unit represents one yard.
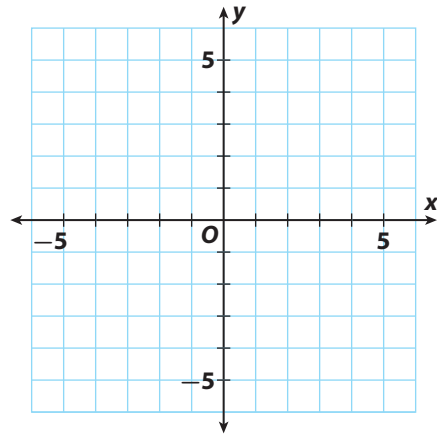
1. Graph the points, and connect them in order. What is the shape of the garden? (Explore Activity)

   _____

2. How much fencing will the gardener need to enclose the garden? (Example 1)

   Each side of the garden is _____ yards in length.

   The gardener will need _____ yards of fencing to enclose the garden.

3. What is the area of the garden? (Example 2)

   _____

4. A clothing designer makes letters for varsity jackets by graphing the letters as polygons on a coordinate plane. One of the letters is polygon *ABCDEF*. The vertices of this polygon are *A*(3, −2), *B*(3, −4), *C*(−3, −4), *D*(−3, 4), *E*(−1, 4), and *F*(−1, −2). Each grid unit represents one inch. Graph the points on the coordinate plane, and connect them in order. Identify the letter formed. Then find its area. (Example 2)

   _____
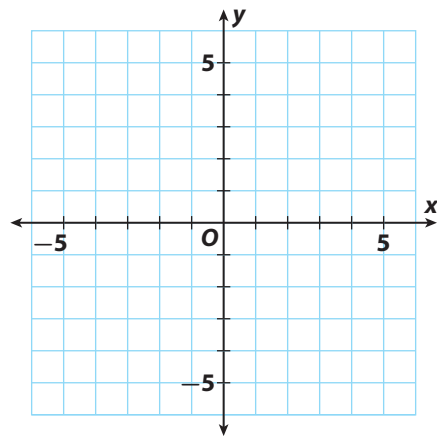
? ESSENTIAL QUESTION CHECK-IN

5. How can you use a coordinate plane to solve perimeter and area problems?

   _____

   _____

   _____

   _____

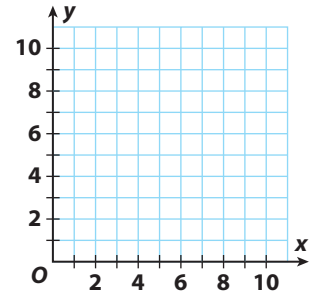## 14.2 Independent Practice

COMMON CORE 6.G.3

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6. A graphic designer creates letters for wall art by first graphing the letters as polygons on a coordinate plane. One of the letters is polygon *MNOPQRSTUV* with vertices *M*(2, 1), *N*(2, 9), *O*(7, 9), *P*(7, 7), *Q*(4, 7), *R*(4, 6), *S*(6, 6), *T*(6, 4), *U*(4, 4), and *V*(4, 1). Each grid unit represents one inch.

a. Graph the points on the coordinate plane, and connect them in

order. What letter is formed? _____

b. The designer will use decorative tape to paint the outline of the

letter on a wall. How many inches of tape are needed? _____

c. How much space does the letter cover on the wall? _____

d. How did you find your answer to c? Use the name(s) of shapes in your answer.

_____

_____

7. Vocabulary The polygon shown is a regular polygon since all sides have equal length and all angles have equal measure.

a. The polygon is a regular _____.

b. What is the perimeter of the polygon? _____

c. A line can divide the figure into two identical four-sided polygons. Each polygon has two bases, and one base is twice the length of the other base. Identify the polygon, and give its perimeter.
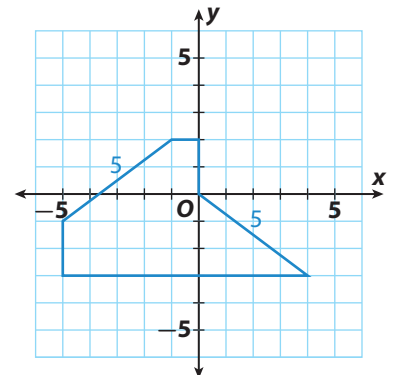
_____
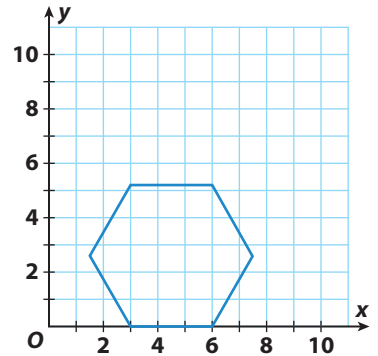
8. Jean wants to put furniture in her clubhouse. She drew a floor plan of the clubhouse, as shown. Each grid unit represents one foot.

a. Which polygon names the shape of the floor?

_____

b. How many feet of baseboard are needed to go around the entire clubhouse?

_____

c. How much carpet is needed for the clubhouse floor?

_____

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9. Persevere in Problem Solving To find the area of triangle *ABC*, Jen first drew a square around the figure. Two sides of the square passed through the points *B* and *C*. The other two sides met at point *A*. Draw Jen's square, and explain how you can use it to find the area of triangle *ABC*.
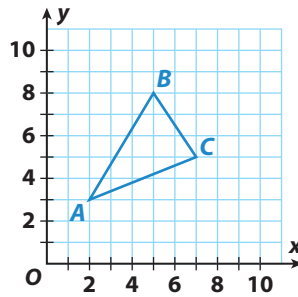

10. Communicate Mathematical Ideas The coordinates $A(5, -2)$, $B(3, -1)$, $C(-4, -4)$, $D(-3, 8)$, and $E(-1, 4)$ form the vertices of a polygon when they are connected in order from *A* through *E*. Classify the polygon without plotting the points. Explain your answer.

11. Explain the Error Josh's teacher draws a regular octagon on a coordinate plane. One side has endpoints at (1, 5) and (4, 5). Josh says he can't find the perimeter of the octagon because he can only find lengths of horizontal and vertical segments. He says he can't find the lengths of the slanted sides of the octagon. What mistake is Josh making? What is the perimeter of the octagon?

12. Critical Thinking Give coordinates for the vertices of a triangle that could have an area of 35 square units. Prove that your triangle fits the description by finding its area.

Ready to Go On?
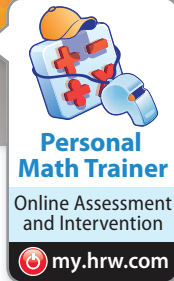
14.1 Distance in the Coordinate Plane

1. Reflect *A* across the *x*-axis. Label the reflection as *N*, and give its coordinates on the graph.

2. Reflect *B* across the *x*-axis. Label the reflection as *M*, and give its coordinates on the graph.

3. The distance between *A* and *N* is _____.

4. Suppose the graph represents a map in which each grid unit equals 1 mile. If a school is located at *B* and a library is located at *N*, what is the distance

between the school and the library? _____
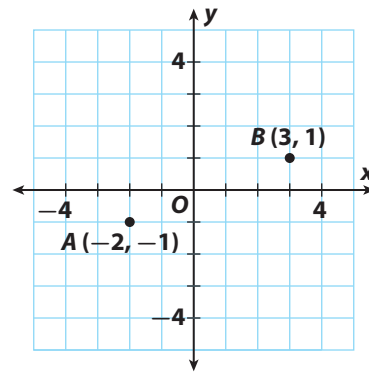
**Find the coordinates of the point for each reflection.**

5. $(-5, 7)$ across the *x*-axis _____

6. $(2, 5.5)$ across the *y*-axis _____

Find the distance between each pair of points.

7. $(1, 1)$ and $(1, -2)$ _____ units

8. $(-2, 3)$ and $(-4, 3)$ _____ units

14.2 Polygons in the Coordinate Plane

9. On the coordinate plane shown, each grid unit represents 10 feet. Polygon *QRST* has vertices $Q(10, 20)$, $R(-10, 20)$, $S(-10, -10)$, and $T(10, -10)$, and represents the floor plan of a room. Find the perimeter and area of the room.

_____

_____
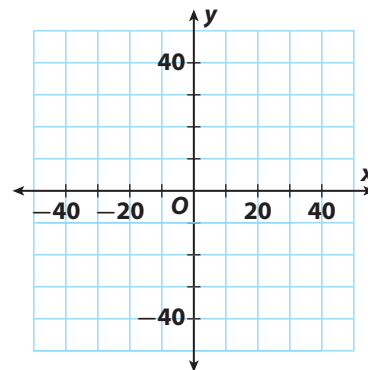
? ESSENTIAL QUESTION

10. Suppose you are given the coordinates of the vertices of a polygon. What steps could you take to solve a problem involving the polygon's area?
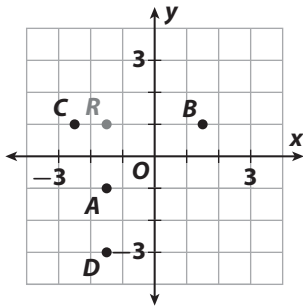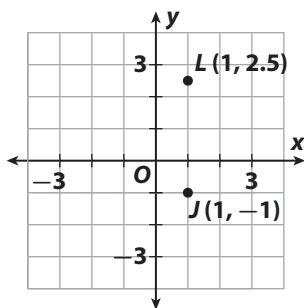
_____

_____

_____

Assessment Readiness

Selected Response

1. Which point is a reflection of point *R* across the *x*-axis?

Ⓐ Point *A*
Ⓒ Point *C*
Ⓑ Point *B*
Ⓓ Point *D*

2. Which point is a reflection of $(12, -8)$ across the *y*-axis on a coordinate plane?

Ⓐ $(-12, -8)$
Ⓒ $(8, 12)$
Ⓑ $(-8, 12)$
Ⓓ $(12, 8)$

3. What is the distance between points *J* and *L* on the grid?

Ⓐ 1.5 units
Ⓒ 3 units
Ⓑ 2 units
Ⓓ 3.5 units

4. What is the greatest common factor of 12 and 30?

Ⓐ 2
Ⓒ 6
Ⓑ 3
Ⓓ 12

5. What is the distance between two points located at $(-6, 2)$ and $(-6, 8)$ on a coordinate plane?
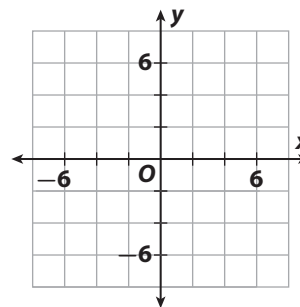
Ⓐ 4 units
Ⓒ 10 units
Ⓑ 6 units
Ⓓ 12 units

6. Which is the sum of $\frac{1}{12} + \frac{3}{8}$?

Ⓐ $\frac{1}{6}$
Ⓒ $\frac{11}{48}$
Ⓑ $\frac{1}{5}$
Ⓓ $\frac{11}{24}$

Mini-Task

7. An artist is laying out the design for a wall hanging on a coordinate plane. She uses polygon *EFGH* with vertices *E*(4, 4), *F*(−4, 4), *G*(−4, −4), and *H*(4, −4) to represent the finished piece. Each unit on the grid represents two feet.

a. Plot the polygon on the grid, and classify its shape.

Name of Polygon: _____

b. How much area will the art cover on a wall? _____