Surface Area and Volume of Solids



Q

ESSENTIAL QUESTION

How can a model help you to solve surface area and volume problems? LESSON 15.1 Nets and Surface Area COMMON 6.G.4

LESSON 15.2

Volume of Rectangular Prisms

LESSON 15.3

Solving Volume Equations 6.EE.7, 6.G.2





my.hrw.com

Go digital with your

write-in student

edition, accessible on

any device.

Real-World Video

Surface area and volume can be important considerations when constructing or repairing buildings or other structures.

my.hrw.com

Math On the Spot

Scan with your smart

phone to jump directly

to the online edition,

video tutor, and more.



Animated Math

Interactively explore key concepts to see how math works.

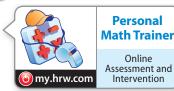


Personal Math Trainer

Get immediate feedback and help as you work through practice sets.

Are **VOU** Ready?

Complete these exercises to review skills you will need for this module.



Use of Parentheses

EXAMPLE	$\frac{1}{2}(14)(12+18) = \frac{1}{2}(14)(30)$	Perform operations inside parentheses first.
	= 7 (30)	Multiply left to right.
	= 210	Multiply again.

Evaluate.

1. $\frac{1}{2}(3)(5+7)$ **2.** $\frac{1}{2}(15)(13+17)$ **3.** $\frac{1}{2}(10)(9.4+3.6)$ **4.** $\frac{1}{2}(2.1)(3.5+5.7)$

Area of Squares, Rectangles, Triangles

EXAMPLE Find the area of the rectangle. 8 ft 3 ft 3 ft 4 = bhUse the formula for area of a rectangle. $= 8 \cdot 3$ Substitute for base and height. = 24Multiply. Area equals 24 square feet.

Find the area of each figure.

5. a triangle with base 6 in. and height 3 in.

6. a square with sides of 7.6 m

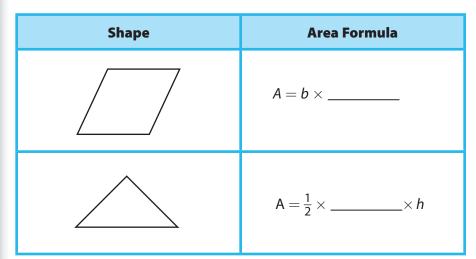
7. a rectangle with length $3\frac{1}{4}$ ft and width $2\frac{1}{2}$ ft _____

8. a triangle with base 8.2 cm and height 5.1 cm _____

Reading Start-Up

Visualize Vocabulary

Use review words to complete the graphic.



Vocabulary

Review Words

area (área) base (base) height (altura) rectangular prism (prisma rectangular) volume (volumen)

Preview Words

net (*plantilla*) pyramid (*pirámide*) surface area (*área total*)

Understand Vocabulary

Complete the sentences using the preview words.

1. The total area of all the faces of a three-dimensional figure is

called the _____.

2. A model that looks like an unfolded three-dimensional figure is a

3. A three-dimensional shape with a polygon for a base and triangles

for sides is a ______.

Active Reading

Booklet Before beginning the module, create a booklet to help you learn the concepts in this module. Write the main idea of each lesson on each page of the booklet. As you study each lesson, write important details that support the main idea, such as vocabulary and important steps in solving problems. Refer to your finished booklet as you work on assignments and study for tests.



MODULE 15 Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

COMMON 6.G.2

Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = lwh and V = bh to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

What It Means to You

You will use the formula for the volume of a rectangular prism.

UNPACKING EXAMPLE 6.G.2

Jala has an aquarium in the shape of a rectangular prism. The dimensions of the base of the aquarium are $1\frac{1}{4}$ feet by $\frac{1}{2}$ foot, and the height is $\frac{3}{4}$ foot. Find the volume of the aquarium.

 $v = l \cdot w \cdot h$ $= 1\frac{1}{4} \cdot \frac{1}{2} \cdot \frac{3}{4}$ $= \frac{5}{4} \cdot \frac{1}{2} \cdot \frac{3}{4}$ $= \frac{15}{32}$



The volume of the aquarium is $\frac{15}{32}$ cubic foot.

COMMON 6.G.4

Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures....

What It Means to You

You will use a net to find the surface area of a square pyramid.

UNPACKING EXAMPLE 6.G.4

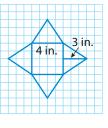
Meg drew a net to find the surface area of a square pyramid.

Square face: $A = b \times h = 16$ square inches

Triangular face: $A = \frac{1}{2}b \times h = 6$ square inches

Total of the areas: $16 + (4 \times 6) = 40$ square inches

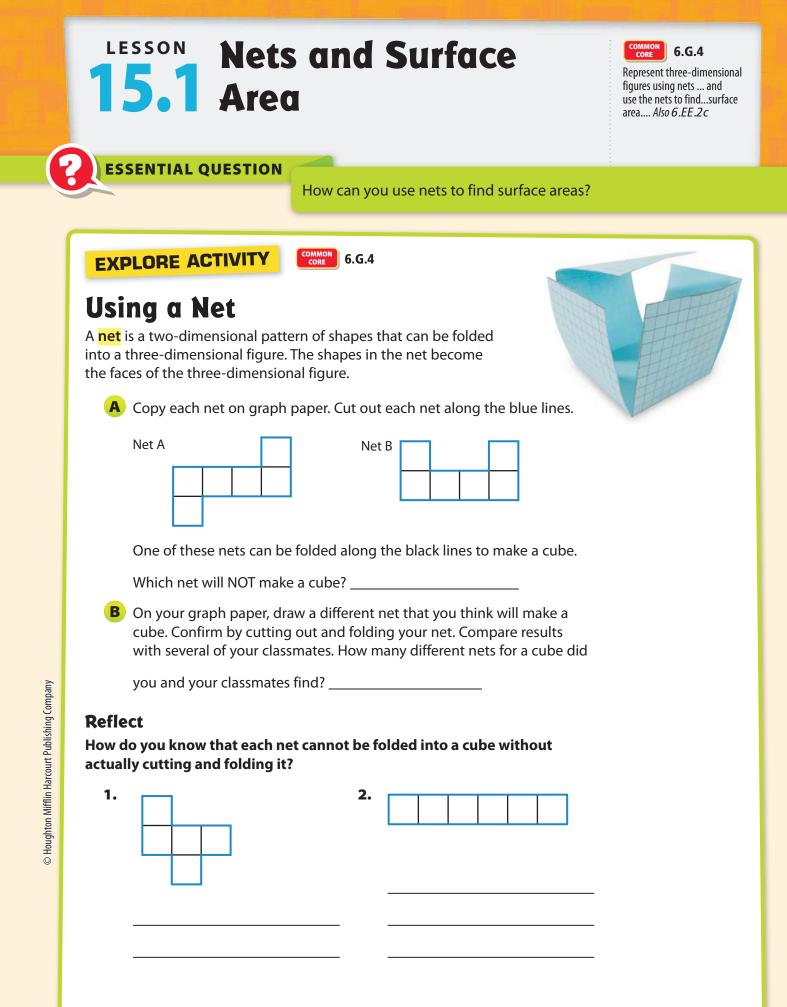
The surface area is 40 square inches.





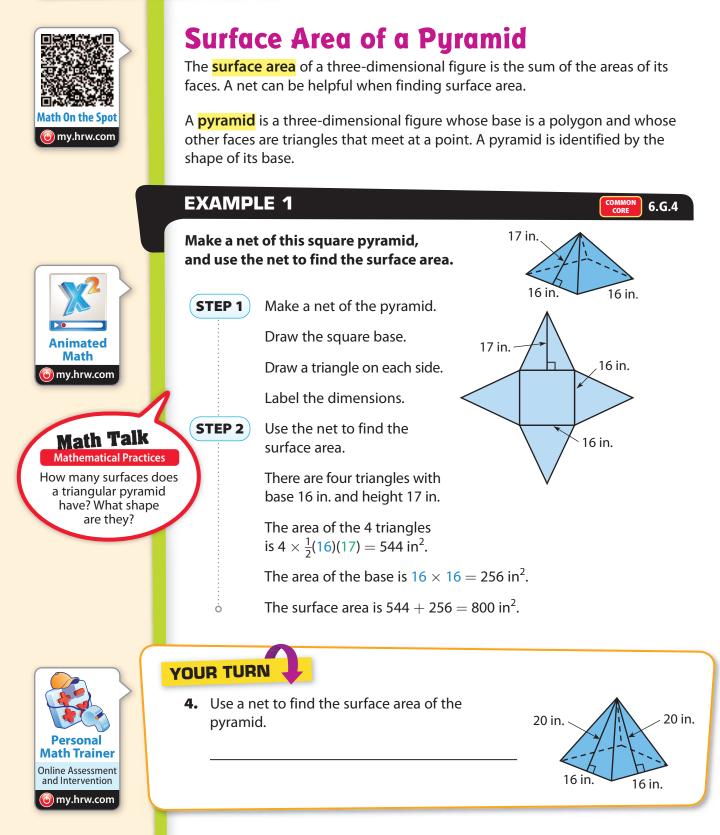
Visit **my.hrw.com** to see all the **Common Core Standards** unpacked.

🙆 my.hrw.com



EXPLORE ACTIVITY (cont'd)

3. What shapes will appear in a net for a rectangular prism that is not a cube? How many of these shapes will there be?



Surface Area of a Prism

A **prism** is a three-dimensional figure with two identical and parallel bases that are polygons. The other faces are rectangles. A prism is identified by the shape of its base.

EXAMPLE 2

A sculpture sits on pedestal in the shape of a square prism. The side lengths of a base of the prism are 3 feet. The height of the prism is 4 feet. The museum director wants to cover all but the underside of the pedestal with foil that costs \$0.22 per square foot. How much will the foil cost?



СОМ

6.G.4

STEP 1 Use a net to show the faces that will be covered with foil.

Draw the top.

Draw the faces of the prism that are connected to the top.

You don't need to include the bottom of the pedestal.

Front

STEP 2

Use the net to find the area that will be covered with foil.

side

Area of top = $3 \cdot 3 = 9 \text{ ft}^2$

The other four faces are identical.

Area of four faces = $4 \cdot 3 \cdot 4 = 48 \text{ ft}^2$

Area to be covered = 9 + 48 = 57 ft²

STEP 3 Find the cost of the foil.

57 · \$0.22 = \$12.54

The foil will cost \$12.54.

Reflect

ò

5. Critical Thinking What shapes would you see in the net of a triangular prism?



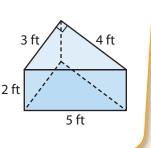
My Notes

•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	
•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	
•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	
•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	
•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•		
•	•	•	•		•	•	•	•	•	•	•	•	•	•	•	•	•		•		•	•		•	•	•	•		
•	•	•	•		•	•	•	•	•		•	•	•	•	•		•	•	•	•	•	•	•	•	•	•	•		
	•	•	•		•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•		
	•	•				•	•	•	•		•						•							•					
								•	•			•							•							•	•		
	•	•			•	•	•	•	•	•		•	•	•	•				•	•	•	•	•			•	•		
																													•

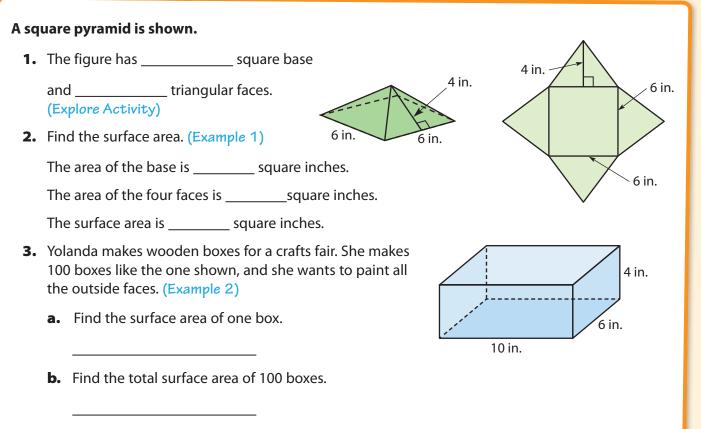


YOUR TURN

6. The figure shown is a triangular prism. How much would it cost to cover the bases and the other three faces with foil that costs \$0.22 per square foot?



Guided Practice

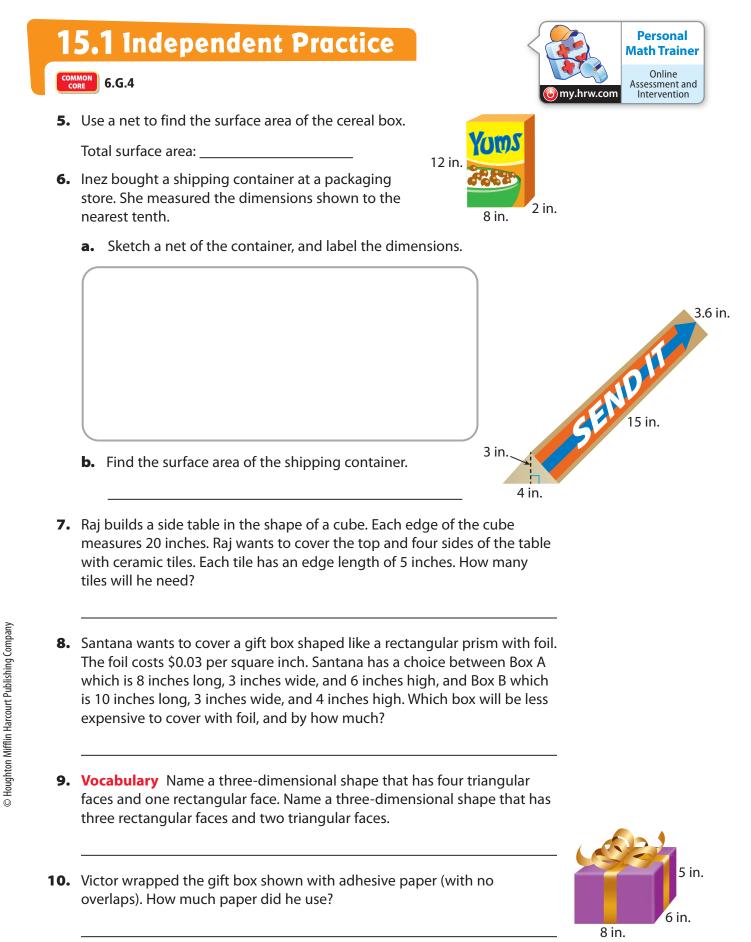


c. One can of paint will cover 14,000 square inches. How many cans of paint will Yolanda need to buy?

ESSENTIAL QUESTION CHECK-IN

4. How is a net useful when finding the surface area of prisms and pyramids?

Class_



Lesson 15.1 **423**

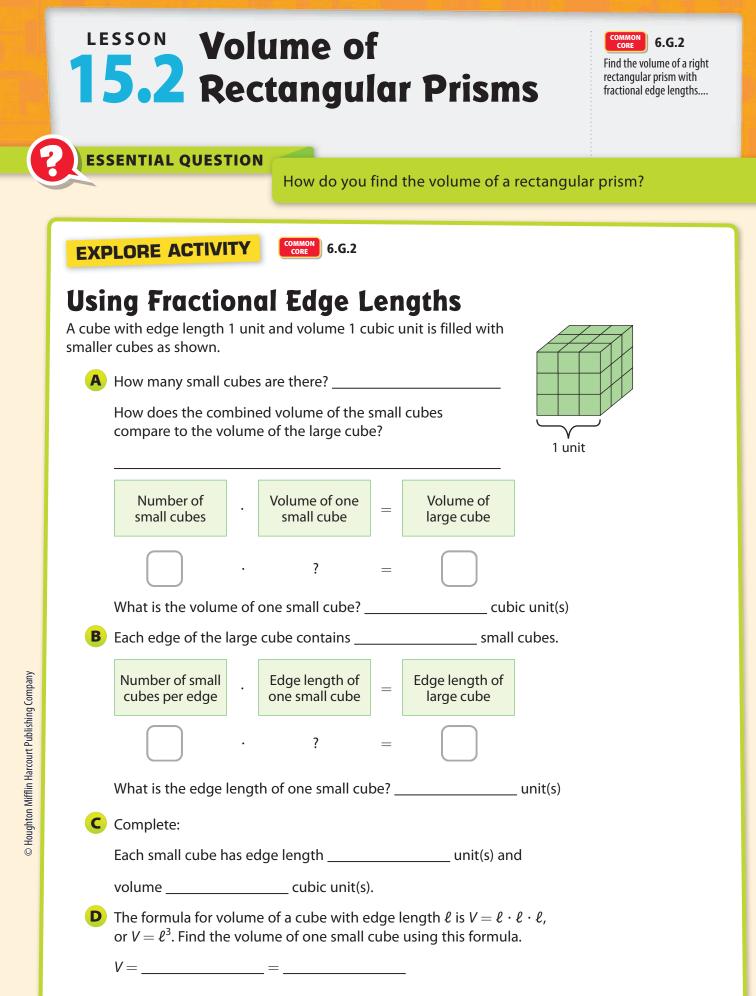
11. Communicate Mathematical Ideas Describe how you approach a problem involving surface area. What do you do first? What are some strategies you can use?

FOCUS ON HIGHER ORDER THINKING

- **12.** Persevere in Problem Solving A pedestal in a craft store is in the shape of a triangular prism. The bases are right triangles with side lengths of 12 centimeters, 16 centimeters, and 20 centimeters. The store owner wraps a piece of rectangular cloth around the pedestal, but does not cover the identical bases of the pedestal with cloth. The area of the cloth is 192 square centimeters.
 - **a.** What is the distance around the base of the pedestal? How do you know?
 - **b.** What is the height of the pedestal? How did you find your answer?

13. Critique Reasoning Robert sketches two rectangular prisms, A and B. Prism A's side lengths are 5 centimeters, 6 centimeters, and 7 centimeters. Prism B's side lengths were twice those of prism A's: 10 centimeters, 12 centimeters, and 14 centimeters. Robert says the surface area of prism B is twice the surface area of prism A. Is he correct? If he is not, how many times as great as prism A's surface area is prism B's surface area? Show your work.

Work Area



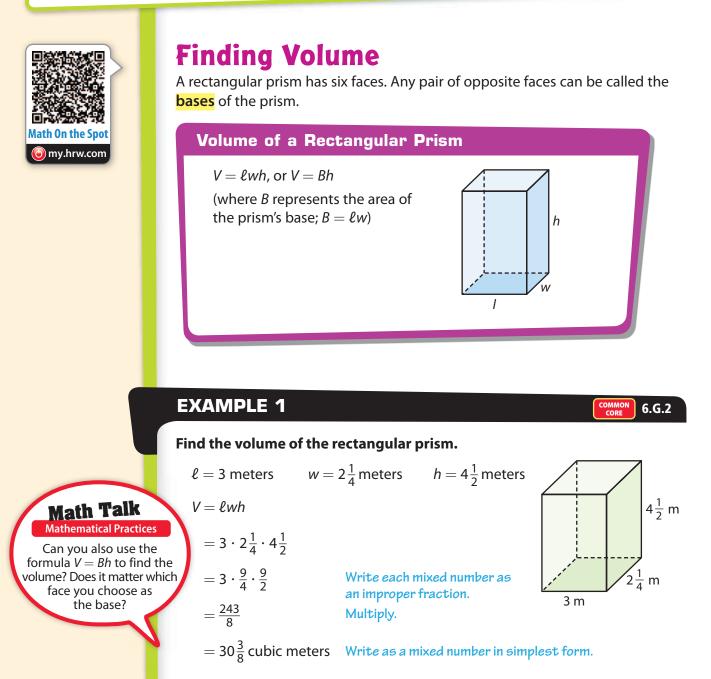
EXPLORE ACTIVITY (cont'd)

Reflect

1. Several of the small cubes in the Explore Activity are arranged into a medium-sized cube as shown.

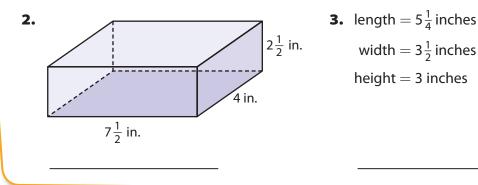


Show two different ways to find the volume of this cube.





Find the volume of each rectangular prism.



3. length = $5\frac{1}{4}$ inches

height = 3 inches

COMMON CORE

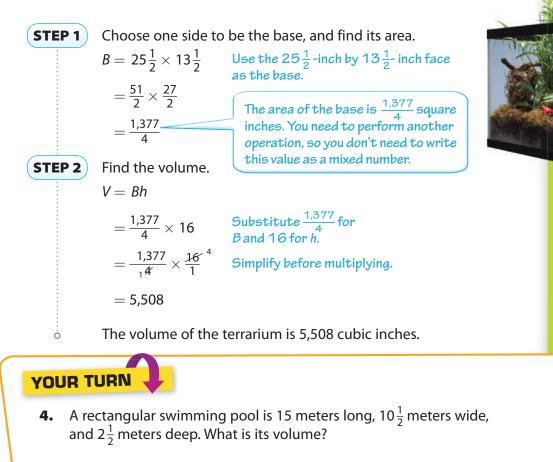
6.G.2

Solving Volume Problems

When you solve a real-world problem involving the volume of a prism, you can choose to use either of the volume formulas you know.

Real EXAMPLE 2

A terrarium is shaped like a rectangular prism. The prism is $25\frac{1}{2}$ inches long, $13\frac{1}{2}$ inches wide, and 16 inches deep. What is the volume of the terrarium?



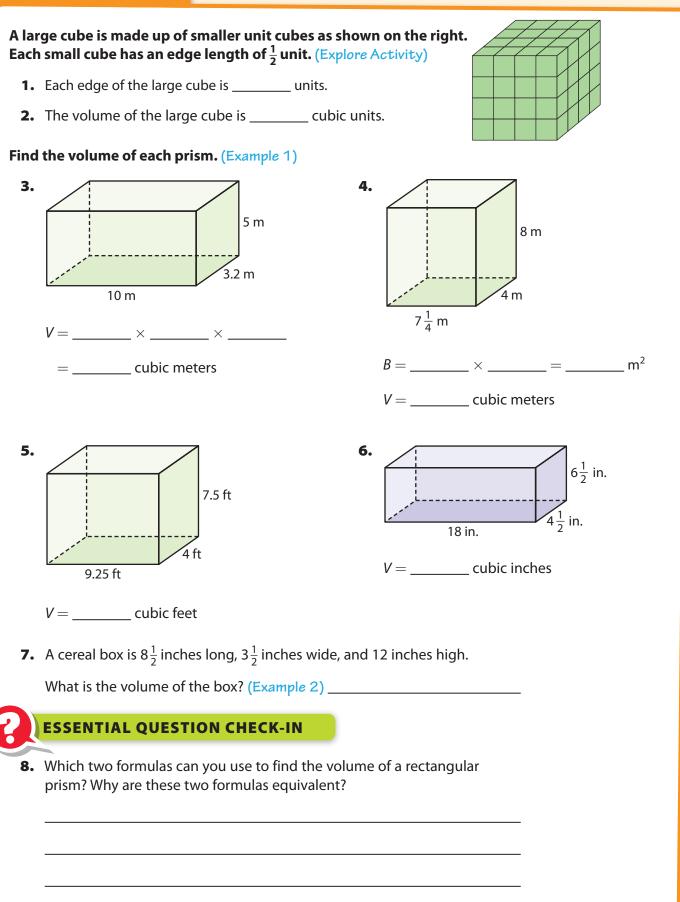
Math Trainer

Online Assessment and Intervention my.hrw.com

Math On the Spo

my.hrw.com

Guided Practice



15.2 Independent Practice

- **9.** A block of wood measures 4.5 inches by 3.5 inches by 7 inches. What is the volume of the block of wood?
- **10.** A restaurant buys a freezer in the shape of a rectangular prism. The dimensions of the freezer are shown. What is the volume of the freezer?
- **11.** Rectangular prism A measures 6 inches by 4 inches by 5 inches. Rectangular prism B's dimensions are twice those of prism A. Find the volume of each prism. How many times as great is prism B's volume as prism A's volume?
- **12.** Leticia has a small paper weight in the shape of a rectangular prism. The dimensions of the paper weight are shown. What is the volume of the paper weight?
- **13.** A company is designing a juice box. The box is in the shape of a rectangular prism. The base of the box is $6\frac{1}{2}$ inches by $2\frac{1}{2}$ inches, and the box is 4 inches high. If juice fills 90% of the box's volume, find the volume of juice in the box.
- **14.** Science Density is the amount of mass in a certain volume of an object. To find the density in grams per cubic centimeter of a substance you

can use this relationship:

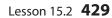
 $Density = \frac{mass in grams}{volume in cubic centimeters}$

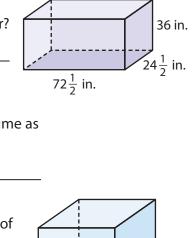
A gold bar that is 16 centimeters by 2.5 centimeters by 5 centimeters has a density of 19.3 grams per cubic centimeter. What is the mass of the gold bar?

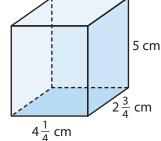
15. A suitcase is a rectangular prism whose dimensions are $1\frac{1}{4}$ feet by

C Houghton Mifflin Harcourt Publishing Company • Image Credits:

 $1\frac{3}{4}$ feet by $1\frac{1}{4}$ feet. Find the volume of the suitcase.









401

a. A moving van rents for \$94.50 per day, and a small truck rents for \$162 per day. Based on the amount of space inside the van or truck, which is the better deal? Explain your answer.

b.	How much greater is the volume of the large truck than the
	volume of the small truck?

c. The family estimates that they need about 1,100 cubic feet to move their belongings. What should they rent?

FOCUS ON HIGHER ORDER THINKING

- **17.** Persevere in Problem Solving A cube has a volume of $\frac{1}{512}$ cubic meter. What is the length of each side of the cube? Explain your thinking.
- **18.** Communicate Mathematical Ideas Think about two rectangular prisms, one labeled prism *P* and one labeled prism *Q*.
 - **a.** Suppose the bases of the prisms have the same area, but the height of prism *Q* is twice the height of prism *P*. How do the volumes compare?
 - **b.** Suppose the area of the base of prism *Q* is twice the area of the base of prism *P*. How do the volumes compare?

19. Critical Thinking The dimensions of a rectangular prism are $3\frac{1}{4}$ feet by $2\frac{1}{2}$ feet by 5 feet. Lee found the volume by multiplying $12\frac{1}{2}$ by $3\frac{1}{4}$. Lola found the volume by multiplying $16\frac{1}{4}$ by $2\frac{1}{2}$. Who is correct? Explain.

Work Area

Inside Dimensions of Trucks Type Length Width Height (ft) (ft) (ft) $10\frac{1}{2}$ Van 6 6 Small $6\frac{3}{4}$ 12 8 Truck Large $8\frac{3}{4}$ $8\frac{1}{2}$ 20 Truck

Solving Volume 15.3 Equations



... Apply the formulas $V = \ell wh$ and V = bh... in the context of solving real-world and mathematical problems. *Also 6.EE.7*

ESSENTIAL QUESTION

How do you write equations to solve problems involving volume of rectangular prisms?

Writing Equations Using the Volume of a Rectangular Prism

You can use the formula for the volume of a rectangular prism to write an equation. Then solve the equation to find missing measurements for a prism.

COMMON 6.G.2, 6.EE.7



EXAMPLE 1 (Real World

Samuel has an ant farm with a volume of 375 cubic inches. The width of the ant farm is 2.5 inches and the length is 15 inches. What is the height of Samuel's ant farm?

V=ℓwh	Write the formula.			
$375 = 15 \cdot 2.5 \cdot h$	Use the formula to write an equation.			
375 = 37.5 <i>h</i>	Multiply.			
$\frac{375}{37.5} = \frac{37.5h}{37.5}$	Divide both sides of the equation by 37.5.			
10 = <i>h</i>				



The height of the ant farm is 10 inches.

Reflect

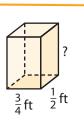
YOUR TURN

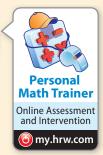
1. Communicate Mathematical Ideas Explain how you would find the solution to Example 1 using the formula V = Bh.

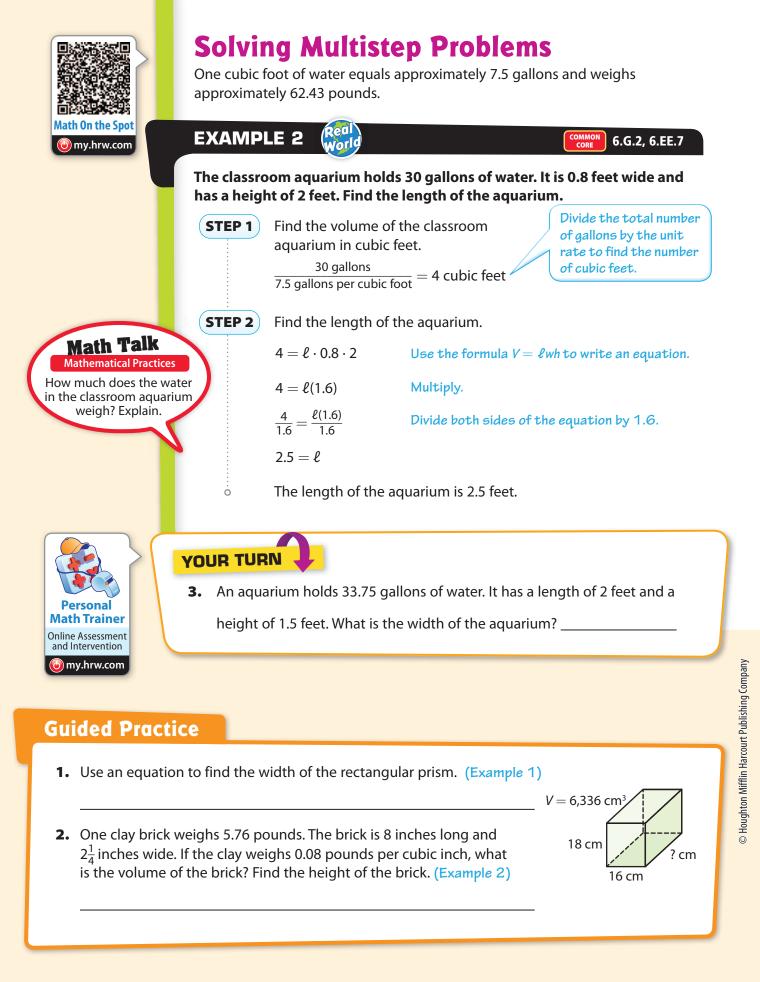
CORBIS

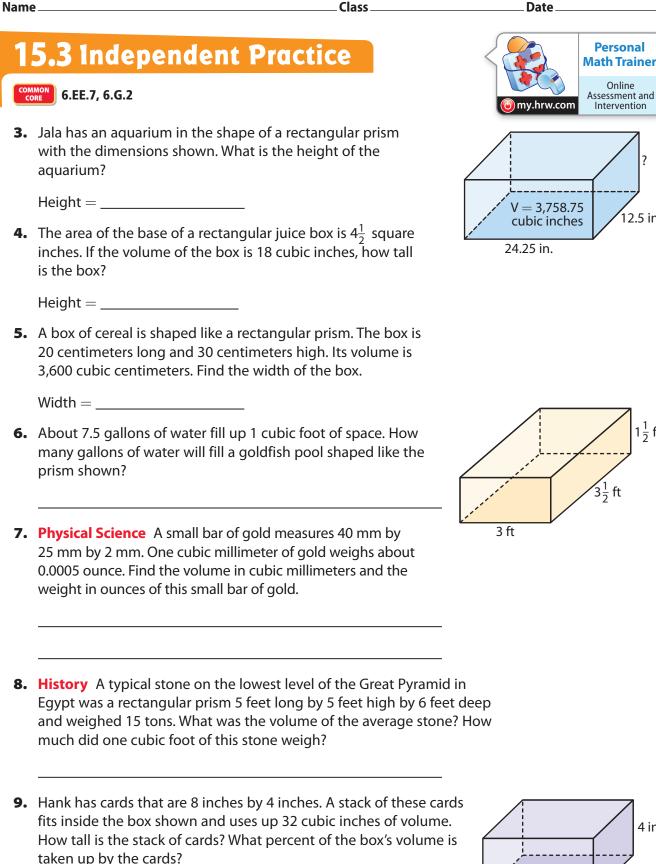


2. Find the height of this rectangular prism, which has a volume of $\frac{15}{16}$ cubic feet.









Lesson 15.3 433

8 in.

4 in.

4 in.

12.5 in.

 $1\frac{1}{2}$ ft

- **10.** A freshwater fish is healthiest when there is at least 1 gallon of water for every inch of its body length. Roshel wants to put a goldfish that is about $2\frac{1}{2}$ inches long in her tank. Roshel's tank is 7 inches long, 5 inches wide, and 7 inches high. The volume of 1 gallon of water is about 231 cubic inches.
 - a. How many gallons of water would Roshel need for the fish? _____
 - b. What is the volume of Roshel's tank? _____
 - c. Is her fish tank large enough for the fish? Explain.



FOCUS ON HIGHER ORDER THINKING

11. Multistep Larry has a clay brick that is 7 inches long, 3.5 inches wide, and 1.75 inches thick, the same size as the gold stored in Fort Knox in the form of gold bars. Find the volume of this brick. If the weight of the clay in the brick is 0.1 pound per cubic inch and the weight of the gold is 0.7 pound per cubic inch, find the weight of the brick and the gold bar. Round all answers the nearest tenth.

Volume of the brick or bar = _____ cubic inches

Weight of the brick = _____ pounds

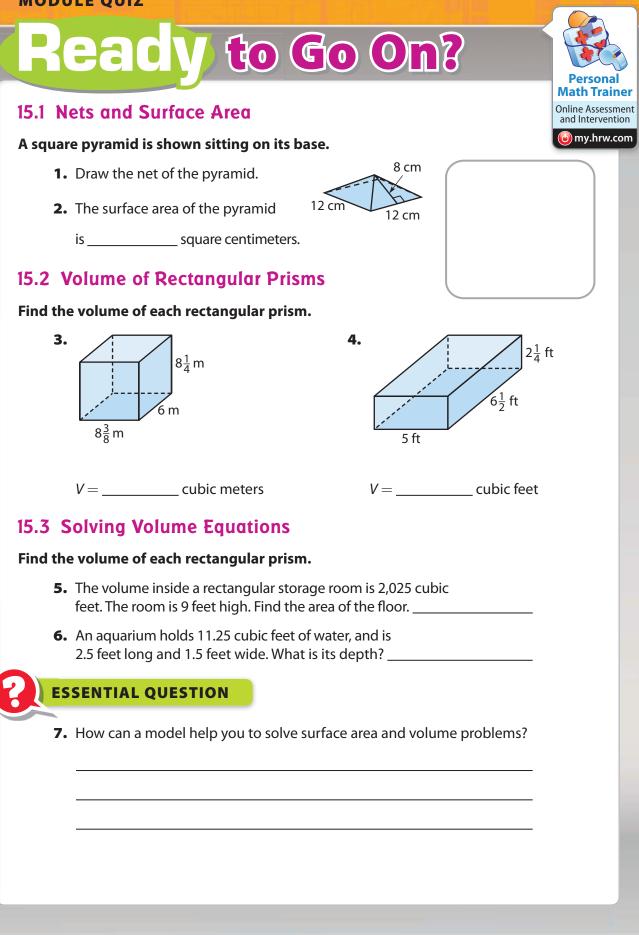
Weight of the gold bar = _____ pounds

12. Represent Real-World Problems Luisa's toaster oven, which is in the shape of a rectangular prism, has a base that is 55 cm long by 40 cm wide. It is 30 cm high. Luisa wants to buy a different oven with the same volume but a smaller length, so it will fit better on her kitchen counter. What is a possible set of dimensions for this different oven?

- **13.** Multiple Representations Use the formula V = Bh to write a different version of this formula that you could use to find the area of the base *B* of a rectangular prism if you know the height *h* and the volume *V*. Explain what you did to find this equation.
- **14.** Communicate Mathematical Ideas The volume of a cube is 27 cubic inches. What is the length of an edge? Explain.

Work Area

MODULE QUIZ



C Houghton Mifflin Harcourt Publishing Company

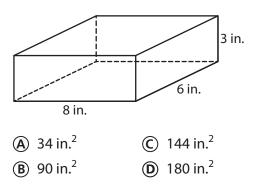


Assessment Readiness



Selected Response

1. Indira is wrapping the box below. How much wrapping paper does she need?



Colin has an ice cube tray with 12 identical compartments. Each compartment is a prism that is 4 centimeters long, 3 centimeters wide, and 3 centimeters high. Given that 1 cubic centimeter holds 1 milliliter of water, how many milliliters of water can the tray hold?

A 36 mL	🔘 432 mL
---------	----------

- (B) 66 mL (D) 792 mL
- **3.** A store manager set up a cardboard display to advertise a new brand of perfume. The display is a square pyramid whose base is 18 inches on each side. The height of each triangular face of the pyramid is 12 inches. How much cardboard was used to make the display?
 - (A) 516 in²
 (C) 756 in²
 (B) 612 in²
 (D) 1,080 in²
- **4.** Which expression is equivalent to 24 + 32?
 - (A) 8 × (3 + 4)
 - **B** 8 × (3 + 32)

(D) 6 × (4 + 6)

5. A bathtub in the shape of a rectangular prism is 5 feet long, $3\frac{1}{2}$ feet wide, and $4\frac{1}{4}$ feet high. How much water could the tub hold?

(A) $14\frac{7}{8}$ ft ³	(C) $74\frac{3}{8}$ ft ³
1 .	1 0

- **(B)** $25\frac{1}{2}$ ft³ **(D)** $87\frac{1}{2}$ ft³
- **6.** The point (-1.5, 2) is reflected across the *y*-axis, What are the coordinates of the point after the reflection?

(A) (−1.5, −2)	€ (2, −1.5)

(B) (1.5, 2) **(D)** (2, 1.5)

Mini-Task

- 7. An cardboard box is open at one end and is shaped like a square prism missing one of its square bases. The volume of the prism is 810 cubic inches, and its height is 10 inches.
 - a. What is the length of each side of

the base? _____

b. Draw a net of the box.

c. How much cardboard is used for the box?